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Problem C: Chemical Espionage

Executive Summary

1. Problem Statement

Ideally, male *Pieris brassicae* butterflies will release anti-aphrodisiac to dissuade other male butterflies from mating with female butterflies without any repercussions and have healthy eggs. In reality *Trichogramma* wasps are able to detect the anti-aphrodisiac and lay their eggs in the butterfly eggs reducing the butterfly population and increasing their own. If the butterflies don't use anti-aphrodisiac, the chance of passing genes reduces; however, if they use too much anti-aphrodisiac, the wasps can detect it and hijack the eggs.

Objective: Develop a mathematical model to describe how the interactions between the *Pieris brassicae* butterflies and *Trichogramma* wasps affect their respective populations over time.

2. Requirements

I. For the sake of this analysis, we will be comparing the populations of the *Pieris brassicae* butterfly and *Trichogramma* wasp.
II. We will start with an initial population of 550 million *Pieris brassicae* butterflies and 250 million *Trichogramma* wasps.
III. We want to estimate the population of *Pieris brassicae* butterflies and *Trichogramma* wasps after a certain amount of time.
IV. We will not account for other factors that may affect the butterfly population such as temperature, humidity, and air pollution.

3. Methods

Hypothesis: We postulate that the *Pieris brassicae* butterfly will minimize the use of anti-aphrodisiacs to preserve its population.

We created a set of differential equations that modeled the change in population of both the *Pieris brassicae* butterflies and *Trichogramma* wasps over time (years). Utilizing the MATLAB ODE 45 (Runge-Kutta Method) differential solver, we were able to plot the change in the overall population of both species as shown above. *(See Appendix)*

4. Analysis

We were able to create a predator-prey differential model of how the population of *Pieris brassicae* butterflies and *Trichogramma* wasps changed by assuming the initial populations of each species. Taking into consideration the data of the interactions between the *Pieris brassicae* butterflies and *Trichogramma* wasps, we were able to estimate the birth rate and death rates of the two species. We then proceeded to program the set of differential equations into MATLAB. Utilizing the ODE 45 solver, we were able to solve our differential equations and plot the results on a graph. The red curve represented the change in the butterfly population over time and the black curve represented the change in the wasp population over time. Our results indicated an oscillating pattern when it came to the change in the butterfly and wasp populations respectively. Initially, the population of butterflies was 550 million and the population of wasps was 250 million. After 100 years, the butterfly population would slightly decrease to 544 million and the wasp population would slightly increase to 254 million. Our results indicate a steady balance in the population over time for both species. (See Appendix)

5. Conclusion

To sum things up, we were able to develop a mathematical model that described how the interactions of the *Pieris brassicae* butterflies as well as *Trichogramma* wasps affected the population of both the butterflies and wasps over time in years. By releasing less anti-aphrodisiacs, the male butterflies were able to hide the scent from the wasps and minimize their influence on the population. Our results indicated that the population of the *Pieris brassicae* butterflies and *Trichogramma* wasps oscillated, but it remained somewhat constant over time. The presence of *Trichogramma* wasps and many other environmental factors such as the temperature, humidity, and pollution play an important role in controlling the population of the *Pieris brassicae* butterfly.

References

Huigens, E., M., Woelke, B., J., Bukovinszky, Smid, … E., N. (2010, February 11). Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps. Retrieved from https://academic.oup.com/beheco/article/21/3/470/219121.

Appendix

1. MATLAB Code and Model

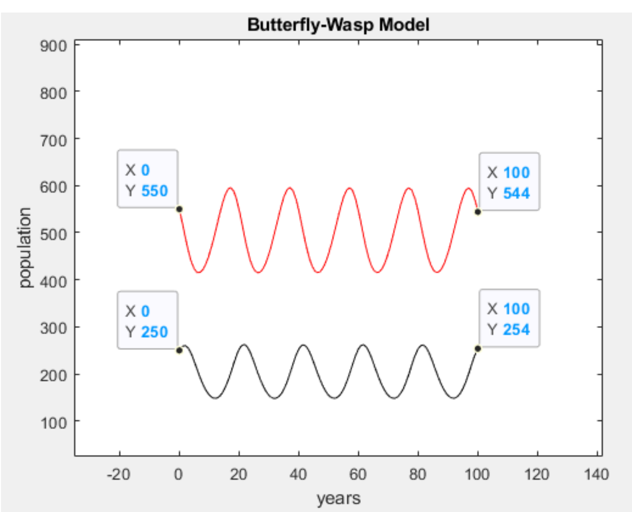
```
% Pieris brassicae butterfly and Trichogramma wasp
% dB/dt=k1B*B(t)-k2B*B(t)W(t), dW/dt= k1W*W(t)*B(t)-k2W*W(t)
% use B(t)= x(1)
% use W(t) = x(2)
% use k1B = 0.2 (birth rate of butterflies per year)
% use k1W = 0.001 (birth rate of wasps per year)
% use k2B = 0.001 (death rate of butterflies per year)
% use k2W = 0.5 (death rate of wasps per year)

function dxdt = butteqn2(t,x)
    dxdt = [0.2*x(1)-0.001*x(1)*x(2);0.001*x(2)*x(1)-0.5*x(2)];
end

% butterfly-wasp analysis

tspan = [0 100]
y0 = [550,250]

[t,y]=ode45(@butteqn2,tspan,y0);
disp(max(y))
%plot(y(:,1),y(:,2),'ro-')
plot(t,y(:,1),'r-',t,y(:,2),'k-')
```



LEGEND		
Symbol	**Description**	**Units**
B	Population of *Pieris brassicae* butterflies	Butterflies
W	Population of *Trichogramma* wasps	Wasps
t	Time	Year
$k1_B$	Birth Rate of *Pieris brassicae* butterflies	Butterflies/Year
$k2_B$	Death Rate of *Pieris brassicae* butterflies	Butterflies/Year
$k1_W$	Birth Rate of *Trichogramma* wasps	Wasps/Year
$k2_W$	Death Rate of *Trichogramma* wasps	Wasps/Year

Red = Change in *Pieris brassicae* butterfly population

Black = Change in *Trichogramma* wasp population