

Problem Statement

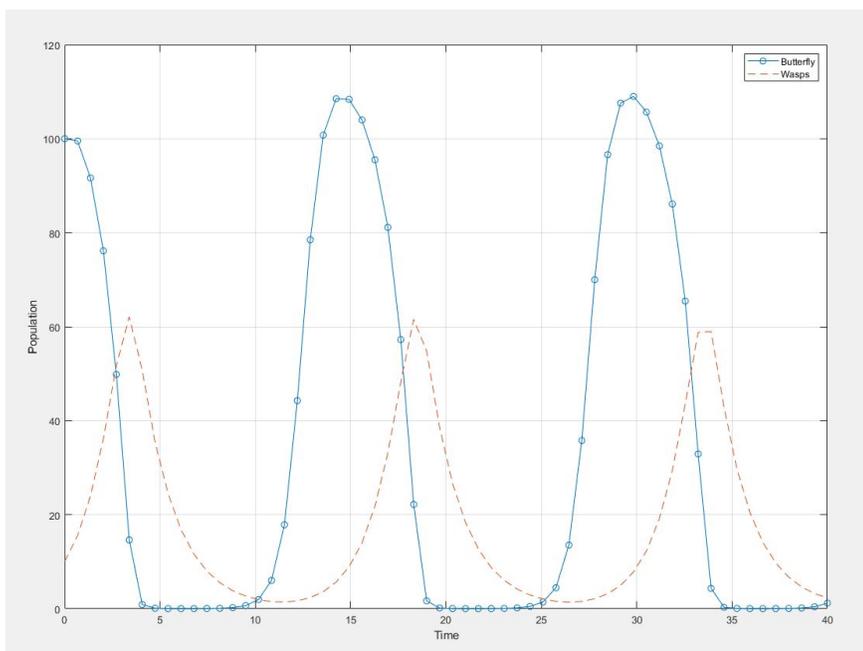
When female butterflies are looking mate, they release chemical signals. These signals are used to attract male butterflies, but it can attract multiple male butterflies. Since there could be multiple males that are attracted by the signal the females release, males use a chemical signals called anti-aphrodisiacs to prevent other males from coming. The problem is that the anti-aphrodisiacs that the males release attracts parasitic wasps. These wasps are a problem for the butterflies because they lay larvae eat the eggs laid by the female butterflies.

We are going to create a mathematical approach in order to model the interaction between the butterflies and wasps caused by the anti-aphrodisiacs released by the males. We must consider; that these chemicals help increase the chance that males fertilize eggs, allow female butterflies to remain unbothered when looking for a suitable place to lay her eggs, but also attract wasp who eat the eggs.

Requirements

For the Chemical Espionage problem we are required to model the interactions between the male and female *Pieris brassicae*, as well as the wasps. We are also required to find out the trade-offs and balance under the influence of anti-aphrodisiacs between these two species (butterflies and wasps). As such, we use differential equations in order to model the relationship between the populations of butterflies and wasps.

Methods (Math Model/Simulation)



$$1. \frac{dx}{dt} = (Ax(1-(x/k))-Bxy/(H+x))-Cx$$

$$2. \frac{dy}{dt} = ((Dxy-(xy^2))/(H+x))-Ey$$

Analysis

Our model is based on the Lotka-Volterra models for predator prey interaction, we modified it by adding logistic type of growth, adding compensation for low prey population and a natural death to simulate a more realistic population change.

Our model has these assumptions:

1. There are no other predators to the butterfly population
2. The only way butterflies interact is mating
3. Both species have a set death rate
4. The anti-aphrodisiac increases the birth rate of butterflies
5. The anti-aphrodisiac increases the predation rate
6. There is no migration by any group

Our model has these Constants:

A-Birth rate of butterflies

B-Predation rate

C-Death rate of butterflies

D-Birth rate of wasps

E-Death rate of wasps

K-Carrying Capacity

H-prey consumption balance term

Conclusion

From our approach, the cyclical nature of our model makes it so that the system repeats itself ad infinitum. That is what happens in the long run, the model has 2 equilibrium points, one of them is extinction the other is a stationary point. Due to some limitations, our model isn't exactly one to one with nature. However given the assumptions we have made, the model does it's best to predict the change in population

References

- 1) Lotka-Volterra Equations. (n.d.). Retrieved from
<http://mathworld.wolfram.com/Lotka-VolterraEquations.html>.