Modeling the Interaction Between Populations of Butterflies and Parasitic Wasps

Members: Max Fomin, Donald Chan, Paul Hultgren
Coach: Dr. Lena Feinmann
Host University: College of San Mateo

Problem

Given in the problem description is an overview of the interaction between a population of large cabbage white butterfly and a certain species of parasitic wasps. In order for the butterflies to attract mates, they have adapted to release a chemical known as an aphrodisiac which males are able to detect and attract to. On the other hand, males are also able to release their own version of pheromones known as anti-aphrodisiacs, which repel other males and make it more likely for them to successfully fertilize the female. However, in addition to driving off other males, certain species of parasitic wasps have been found to pick up on these signals, allowing them to track the fertilized females. As the female goes to lay her brood, these wasps are able to follow her and are then able to parasitize her freshly laid eggs, laying their own within the shell of the now-dead butterfly egg. Our group was tasked with creating a differential model for the interactions within this system and to use this to analyze the long term behaviour.

Model

In creating our model we first made various assumptions in order to simplify and idealize the dynamics of the system. Our first assumption is that the populations neither gain nor lose any members from or to outside populations, thus the only changes result from births and deaths of each respective group. In addition to this, it is assumed that the butterfly population possesses no limit to its food supply, and thus left without parasitism it would group at an exponential rate. Furthermore, we assume that every egg that the wasps parasitize becomes a wasp embryo. Finally, we assume that every egg, wasp or butterfly, which is laid will turn into an adult with certainty. Imposing these assumptions, we arrive at our model.

$$\frac{dP_F}{dt} = \frac{dP_M}{dt} = AP_F + BP_M - CP_M P_F - D(AP_F + BP_M)P_M W \quad \text{(Eq. 1)}$$

$$\frac{dW}{dt} = D(AP_F + BP_M)P_M W - EP_F W \quad \text{(Eq. 2)}$$

P_F is the number of butterfly females
P_M is the number of butterfly females
W is the number of wasps
$A,\ B,\ C,\ D,\ E$ are positive real constants which describe the interaction between the two species

For the butterflies, AP_F accounts for the natural growth of butterflies, which depends proportionally on the number of females. The BP_M accounts for the benefit of male anti-aphrodisiacs, as it allows for females to be bothered less by other males once they have mated. The $-CP_M P_F$ term models the competition both among females in order to find places to lay her eggs, as well as the detriment caused by an excess of male attention.

Finally, the $-D(AP_F + BP_M)P_M W$ is the number of eggs parasitized by the wasps, which depends on the number of eggs laid by the butterflies ($AP_F + BP_M$), the number of butterfly males as this increases the amount of anti-aphrodisiac, and the population of wasps. As it is assumed that the number of eggs parasitized is equal to birthrate of the wasps, this term is again included in (Eq. 2). The second term in (Eq. 2) is $-EP_F W$, which accounts for the competition among the wasps, as it is assumed that parasitism is the wasps main source of food and thus if they are unable to find female eggs they will starve.

Goals

Given a set of values for the constants, we set out to find answers for the following set of questions:

1. How does the initial difference between the female and male populations affect the long term behaviour of the system?
2. Under what circumstances will the populations die out, grow unboundedly, or potentially reach an equilibrium point or cycle?
3. How does the change of certain constants change the behaviour of the system?

Analysis

Due to the number of degrees of freedom imposed by the constants, a comprehensive analysis of the behaviour of this system for all constants is infeasible within the scope of this project. However, there are some general trends which can be found from examining the equations. For example, if the value of D is too low, meaning that the number of eggs parasitized by the wasps is very low, the wasp population will die out and for suitable constants A, B, and C the butterfly population will grow unboundedly, perhaps to a certain carrying capacity which could be included within the model. However, if a suitable value is chosen for all constants such that neither population has the propensity to die, the system exhibits very interesting behaviour, beginning with the wasp population growing at an almost constant rate with the butterfly population decreasing at a constant rate as well. This continues until the wasp population grows large enough and the butterfly population small enough to where there is not enough eggs being laid to sustain the large wasp population. Following this, the number of wasps sharply declines, spiraling towards a point much closer to the origin. This point would be considered a stable fixed point as solutions around it seem to spiral towards it until the rates of each reach zero and equilibrium is reached. From this behaviour we can deduce that this system is only stable for very small populations, and if the populations start very high they will tend towards this equilibrium. This is intuitive as one of our assumptions is that no members of either population are able to leave or join from other groups, and as such it is difficult to sustain a closed system with a large amount of each group if none are allowed to leave.