Slippery Rock University of Pennsylvania
Michael Zirpoli, Jessica Lefler, Noah Quick

The model:

$$B(x,t) = \frac{c}{\frac{L_2}{1 + A_2 e^{-B_2 x}}}\left(1 + K\cos\left(\sqrt{(a_1 x + a_2)c}\,t + \Phi\right)\right)$$

$$W(x,t) = \frac{a_1 x + a_2}{\frac{L_1}{1 + A_1 e^{B_1 x}}}\left(1 + \sqrt{\frac{c}{a_1 x + a_2}}\,K\sin\left(\sqrt{(a_1 x + a_2)c}\,t + \Phi\right)\right)$$

These functions map the butterfly population B and the wasp population W with respect to x, the amount of anti-aphrodisiacs emitted, and t, time. The model came from a derivation of the predator-prey model.

The original predator-prey model assumes that the change in prey population with respect to time is a product of the prey population and a linear function decreasing as the predator population increases. Similarly, the change in predator population with respect to time is a product of the predator population and a linear function increasing as the prey population increases. In the absence of any predators, the prey population is an exponential growth function and in the absence of any prey, the predator population is an exponential decay function.

The first variation we made to the model assumes that the constants will instead be functions with respect to the amount of anti-aphrodisiac chemical emitted by male butterflies. In particular, we assumed that the interaction constant in each of the populations' first time derivative is now a logistic growth function with respect to the amount of anti-aphrodisiac chemical. This allows us to model the interaction between the butterflies and the wasps. We choose an increasing logistic growth as the interaction in the butterfly population because more wasps will be attracted to female butterflies when more chemicals are released, thus reducing the butterfly population; similarly, a decreasing logistic growth was chosen as the interaction in the wasp population because as the amount of chemical released increases, the wasps will be more attracted to females and thus be able to lay more eggs in the butterfly eggs.

The second variation we made to the model was the constant allowing the butterfly population to have an exponential growth in the absence of predators. Specifically, we choose to change this constant into an increasing linear function. The reason behind this change was so that the butterfly population would benefit more from having more chemicals released by males, otherwise the optimal amount of chemical would be 0 and the wasp population would die down.

Since many assumptions were made about each of the constants used throughout the function, each one had to be analyzed to see how changes affected the populations predicted by the function. In the wasp function, as B_1 increases, higher chemical increases are seen to benefit the wasps. Increases in L_1 and a_2 cause significant increases in the population predicted for any given amount of chemical. The values for a_1 and a_2 act similarly to this, but on a smaller scale. Changes in Φ lead to minimal changes in the overarching function.

In the butterfly function, we can observe that even small changes to the value of c can lead to the period of the function becoming smaller and smaller. The values of a_1 and a_2 also significantly decrease the period size, however a_2 only affects populations based on lower chemical levels. B_2 and k tend to work opposite to each other, with B_2 causing the population to decrease at higher chemical levels and k increasing the population at lower chemical levels. The carrying capacity of the butterflies greatly decreases the value of the population at all points as it increases while A_2 only slightly increases the population as it increases. Φ leads to a horizontal shift of the graph to the left (towards lower t values) as it increases in value.

These effects were observed from viewing each constant's position within their respective equations, however, without concrete and experimental data, determining exact values for each of the constants becomes difficult. However, we would expect that in the long run the populations' reproductive periods would be of the same length. With a consistent difference between the two functions as time continues.

References

https://link.springer.com/article/10.1007/s10340-010-0339-9

Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, Nina E. Fatouros, Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking *Trichogramma* wasps, *Behavioral Ecology*, Volume 21, Issue 3, May-June 2010, Pages 470–478, https://doi.org/10.1093/beheco/arq007

Boyce, William E., and Richard C. DiPrima. *Elementary Differential Equations and Boundary Value Problems*. 10th ed., John Wiley & Sons, Inc., 2012.