CHEMICAL ESPIONAGE: A Solution

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Introduction

The male cabbage white butterfly (Pieris brassicae) uses chemical signals, called anti-aphrodisiacs, to help ensure the monogamy of females during a particular mating period. This allows the females to focus more so on selecting a location to build their nests, rather than having to entertain other interested males. However, parasitic wasps are able to detect the anti-aphrodisiac, making it easier for the wasps to locate a nesting female. This increases the probability of predation on the butterfly eggs by wasp larvae. Theoretically, there is an optimal percentage of female butterflies that would need to be affected by the anti-aphrodisiac to maximize the growth rate of the population. Our goal was to find said percentage.

Solution

In general, population growth is proportional to the current population as well as the difference between the carrying capacity and the current population (*K - P)* and can be modeled by the following equation

$$\frac{dP}{dt} = rP(1 - \frac{P}{K})$$

where P is a function of time representing the current population, $\frac{dP}{dt}$ is the current rate of growth in the population, *r* is the rate of increase of the population per capita, and K is the carrying capacity for the given species in a particular ecosystem. Exponential growth is thereby expected, of course. We argue that the rate of increase per capita, *r*, will vary with the number of affected females in the case of the white cabbage butterfly. For example, if no females are affected by the anti-aphrodisiac, they all theoretically would be distracted by competing males for a greater portion of the mating period, making it more difficult to optimize their nests/location of the nests, increasing the probability of predation and consequently lowering the butterfly population growth. Similarly, if all females are affected, there may be competition amongst the females for potentially limited superior nesting locations, or there would be such a high density of nests in what would otherwise be considered advantageous areas that the probability of the nests being found by the wasps would go up, given they are able to detect the anti-aphrodisiac. Therefore, *r* will be a function of *x* given by

$$r(x) = \xi e^{-\beta(a-\frac{x}{p})^2}$$

where *x* represents the number of females affected by the anti-aphrodisiac, *p* represents the current population of mature females, ξ represents the real maximum growth rate per capita, β will be a real number such that $\beta \geq 1$ to expand the lower bound of *r*, and *a* represents the percentage of the given area that is considered to be superior for nesting. Analysis shows that if the percentage of females $\left(\frac{x}{p}\right)$ falls below or above the percentage of area that is advantageous for nesting significantly, the growth rate per capita will fall. The theoretical optimal percentage of affected females should equal *a*, we argue, assuming the significance of the density of nests in a given section of the total area regarding predation. In this case, the real maximum growth rate per capita will be achieved, as $e^{-\alpha^2}$ has a maximum of 1 at $\alpha = 0$.

Other Variables to Consider

- The distribution of advantageous locations in a given area
- Function that models the rate of predation
- Death rate contributing to the rate of growth/decay in the population
- Growth of the wasp population yielding a lower rate of growth in that of the butterflies

The absence of these other potentially significant contributing factors seems to make the solution a bit more trivial of an approximation. Varying wasp population, for example, would have an effect on *r*, specifically on ξ and β which would theoretically decrease and increase respectively with a growing wasp population.

In our solution, the percentage of advantageous area for nesting, represented as *a,* was considered to be a constant. This is true when an area of land has constraints on its perimeter, but another factor to consider would be the distribution of the advantageous area. For example, you could have square surface area with 20% on it representative of superior nesting area and have practically an infinite amount of ways to distribute it. Mathematically speaking, they are all the same at 20%. But in a real world situation, the distribution makes a world of a difference. If the superior area was all located all in the same place versus it being evenly distributed, this could have a huge impact on the growth rate of the population. For a more accurate approach, one should not only consider the percentage of the advantageous area, but how it is distributed throughout the system.

Conclusions

The mathematical model presented accurately predicts the current rate of growth of the population, while also countering for the fluctuation in the rate of increase per capita of the cabbage butterfly population. For even further accuracy, we recommend consideration of the variable for the distribution of the advantageous area, a function depicting the rate of predation and growth rate of the wasp population, and the death rate of the butterflies. The current model is not perfect, but actively attempts to solve the issue at hand.