SCUDEM Executive Summary

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Introduction

Our team approached Problem C, which addressed the mating habits of butterflies in the genus *Pieris*. These butterflies use an anti-aphrodisiac chemical in mating, and parasitoid wasps in the genus *Trichogramma* make use of this chemical to find butterfly eggs (inside of which they lay their eggs). There seems to be a selective pressure for the butterflies to reduce use of the anti-aphrodisiac[1]. Our goal is to accurately model how changes in amount of released anti-aphrodisiac will affect the butterfly population. On one hand, a decrease in the amount of chemical released will yield a worse strategy for the butterflies' mating rituals; on the other hand, it will also decrease the likelihood of a given butterfly's eggs being parasitized. Which of these pressures is more significant?

Assumptions

We assume that *Pieris Brassicae* and *Trichogramma brassicae* do not have a collective seasonal life cycle, but rather mate continuously, and are thus their population is better modeled by differential equations than difference equations. We also assume that the interaction of the butterflies and wasps behaves identically to that of a predator-prey relationship; although the wasps do not kill the adult butterflies, by infecting their spawn, the wasp's parasitism removes a butterfly from the gene pool and removes its offspring from the next generation, which has the same effect as killing the adult. Furthermore, we consider males of both species to be in ample supply for the sake of reproduction;since once male can mate with many females, even if there are fewer males than females, the population will not suffer. Thus, we need only track female butterfly and female wasp populations. We also assume that both the butterfly and wasp have sufficient food and no major environmental limiting factors besides the reliance of the wasp on sabotaging the reproduction of butterflies in order to reproduce on its own. Thus, the Lotka-Volterra model is effective in describing the relationship, since it resembles that of a predator-prey relationship. Lastly, we assume both species thrive year-round in their environment - our model does not account for overwintering habits of either species.

Model

We use the Lotka-Volterra equations to model this interaction. We define variables and constants as follows: x is the butterfly population, y is the wasp population, t is time (measured in butterfly life-cycle lengths, about 2 months), A is the amount of anti-aphrodisiac found on the wing of a mated butterfly, α is the growth coefficient for the number of butterflies, β is the proportion of butterflies whose eggs are parasitized by a given wasp, γ is the proportion of of female wasp spawn that make it to adulthood, and δ is the death coefficient for the number of wasps. We picked the following numbers for the variables, based on research and our intuition for the shape of our model's phase space:

$$A = 2.0 \quad [1]$$

$$\delta = 6$$

Furthermore, we expect both α and β to behave as functions of A, since the presence of an anti-aphrodisiac chemical affects both butterfly mating behaviors and wasp search behavior. We model $\alpha(A)$ as a sigmoid. Our reasoning is as follows: the presence of A increases the chances that a given male butterfly will not waste its time attempting to mate with a mated female. Thus, an increase of A yields a better mating strategy, but only up to a point; once nearly every male butterfly is able to immediately tell that a certain female has been mated with, the addition of more A is unhelpful. In addition, we wanted $\alpha(2.0) = 10$, as research shows that this reflects the

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number of females of every egg clutch that make it to adulthood [2]. Specifically, we used the function

$$\alpha(A) = \frac{10e^{4A-2}}{e^{4A-2} + 1}$$

in our model.

We model $\beta(A)$ as a Gaussian curve. This is because *Trichogramma* wasps are specifically attuned to the amount of anti-aphrodisiac released by the butterflies [1]. Thus we adjusted our curve to peak at $A = 2.0$, approximately the standard amount of A. In addition, we want this peak to reflect the rate of predation we find in the wild, which is often about 90% [3]. We used the function

$$\beta(A) = 0.9e^{-(A-2)^2}$$

to model the ability of the wasps to find mated female butterflies, given the amount of A those butterflies had present on them.

Lastly, we note that the growth rate of the wasps should be proportional to the predation rate, as wasp reproduction is related directly to the rate at which wasps parasitize prey eggs. Thus, we have $\gamma(A) = k \cdot \beta(A)$. We took $k = 0.67$, implying that for every three successful parasitisms, two new adult females would be created. However, it was difficult to find research that supports this choice of k. We did note that the model did not change much from our choice of k - the only non-negligible change was that the wasp population grew proportionally larger at all points for larger k, and thus with more data k could be more accurately fit to a real-life scenario.

We now have the set of parameters A, $\alpha(A)$, $\beta(A)$, $\gamma(A)$, and δ. The rate of change of both the butterfly and wasp population is modeled by their birth rate minus their death rate, i.e.

$$\frac{dx}{dt} = \alpha(A)x - \beta(A)xy$$

and

$$\frac{dy}{dt} = \gamma(A)xy - \delta y = k\beta(A)xy - \delta y,$$

the Lotka-Volterra equations.

Analysis

Using the Runge-Kutta method, we obtained numerical approximations for points of the form (t, x_t, y_t). We found that the Lotka-Volterra model captures much of the expected behavior of this system. For example, both the butterfly and wasp population exhibited periodicity in their populations, with the wasp population peaking slightly after the butterflies do each time. We note that most variables in this model are quite robust, as well: slight adjustments to δ or the parameters in $\alpha(A)$ or $\beta(A)$ create only small variations in the data sets returned from the Runge-Kutta method.

Notably, however, one variable that caused a great deal of change in our data was A. The mere variation of A from 2.0 to 1.8 increased the butterfly population significantly. When A varied from 2.0 to 1.5, the population nearly doubled! This is indicative of (at least) one of two things. Our choices of $\alpha(A)$ and $\beta(A)$ may be too sensitive, or there is an incredibly strong selective pressure for *Pieris* to decrease the amount of anti-aphrodisiac it releases. If the latter is the case, then we have successfully modeled how changes in A affect changes in butterfly population, and it is clear that the pressure to decrease anti-aphrodisiac due to the presence of the wasp population is significantly more pertinent than the pressure to continue to use such a high quantity of anti-aphrodisiac to mate.

We end with two interesting observations: firstly, although unlikely in nature, if the butterflies released *more* anti-aphrodisiac, this would also be a successful model - too much A will fool the wasps, which are looking for a specific amount of the chemical. However, it is unlikely that evolutionary pressures would tend for this to happen [1]. Secondly, when A was decreased to 1.5, the butterfly population skyrocketed (as we noted above). More interestingly, the wasp population rapidly grew too. The decrease of A led to there being more potential breeding grounds for wasps, even though they became less efficient at finding eggs. When you're a predator, perhaps you need to let your prey thrive a little bit, too.

References

[1] M Huigens et al. Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking trichogramma wasps. *Behavioral Ecology*, 21(3):470–478, 2010.

[2] A. D. Le Masurier. Costs and benefits of egg clustering in pieris brassicae. *Journal of Animal Ecology*, 63(3):677–685.

[3] Natural History Museum. Universal chalcidoidea database.