Simiode - Problem C
Chemical Espionage
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Introduction/Overview:

Problem C focuses on the interactions between large white cabbage butterflies, *Pieris Brassicae*, and the two parasitic wasp species *Trichogramma brassicae* and *Trichogramma evanescens*. These wasps are attracted to an anti-aphrodisiac which the male butterflies transfer onto female butterflies after mating. The anti-aphrodisiac increases the chance that a male butterfly fertilizes eggs, and it allows the mated female to spend most of its energy finding a place to lay her eggs. The chemical also increases the chance that a female's eggs will be parasitized by a wasp. In this report, we start by listing important theoretical values which can be approximated via experimentation, then we develop a discrete model and a continuous model that describe the butterfly population as it interacts with the wasp population. We also include possible drawbacks for each model and an educated guess for the long run behavior of the butterfly population [1].

Variables:

- A_0 = initial population
- A_i = population of generation i
- L = number of eggs laid by a female butterfly
  - L is a random variable with the condition $20 \leq L \leq 50$
- $\bar{L}$ = average number of eggs laid by a female butterfly
- W = number of eggs in a clutch that are parasitized by a wasp
  - W is a random variable with the condition $20 \leq W \leq 50$
- $\bar{W}$ = average number of eggs in a clutch that are parasitized by a wasp
- p = probability that a female butterfly is mounted by a wasp
- d = percentage of deaths prior to mating season

Assumptions/Research:

We assume for simplification that both wasp species behave the same, that half the butterfly population at any given time is female, and that female butterflies lay only one clutch of eggs per mating season, and p is constant (in reality p is more likely a function of the wasp population). According to animalspot.net, the female large cabbage white butterfly has 20 to 50 eggs per clutch; also, the species has two to three mating seasons per year.

Modeling:

Our group found that a given generation A_i can be modeled by the recurrence relation $A_i = A_{i-1}(1-d) + \left(\frac{A_{i-1}(1-d)}{2}\right)(\bar{L} - p\bar{W})$ where $A_{i-1}(1-d)$ describes the number of survivors from the previous generation at the beginning of the mating season, $\frac{A_{i-1}(1-d)}{2}$ describes the number of surviving females from the previous generation that lay eggs, and $\bar{L} - p\bar{W}$ describes the expected number of eggs that survive. This model can be factored to get $A_i = A_{i-1}(1-d)\left(1 + \frac{\bar{L}-p\bar{W}}{2}\right)$.Using an initial population of A_0, we see that $A_1 = A_0(1-d)\left(1 + \frac{\bar{L}-p\bar{W}}{2}\right)$ and $A_2 = A_1(1-d)\left(1 + \frac{\bar{L}-p\bar{W}}{2}\right)$ $= A_0(1-d)^2\left(1 + \frac{\bar{L}-p\bar{W}}{2}\right)^2$. From here, we begin to see a pattern in the population of each generation, and our discrete model becomes $A_i = A_0(1-d)^i\left(1 + \frac{\bar{L}-p\bar{W}}{2}\right)^i$.Unfortunately, plugging any set of reasonable parameters will reveal that this model fails to bound the increase of population size, causing the population to skyrocket to unreasonable heights.

Our equation for the discrete case, $A_i - A_{i-1} = rA_{i-1}$, where $r = \frac{1-d}{2}(\bar{L} - p\bar{W}) - d$ is the rate of increase in population size, does not account for the carrying capacity of the butterfly population. To factor in the population's carrying capacity K, we simply change the relation to $A_i - A_{i-1} = rA_{i-1}\left(\frac{K-A_i}{K}\right)$; the added factor slows the rate of increase as the population gets closer and closer to K. A continuous analog follows from this equation, namely $\frac{dA}{dt} = rA\left(\frac{K-A}{K}\right)$. The method of separation of variables gives us the general solution $A(t) = \frac{K}{1+Ce^{-rt}}$. By our initial condition $A(0) = A_0$, we can solve for C to get $C = \frac{K-A_0}{A_0}$; clearly, as t approaches infinity, $A(t)$ approaches K.

Conclusion:

On one hand, the discrete model with the added carrying capacity factor seems superior to the continuous model because it takes into account the finite nature of mating seasons. However, we could not derive a formula which generates the ith mating season population depending only on i. This is the advantage of our continuous model; we were able to derive a formula which generates the population size at any time t that depends only on time. In terms of real-world application, our models likely fall short as they depend upon parameters which could be difficult to measure considering the rarity of the large white cabbage butterfly. If experimentation discovers accurate measurements for our parameters in the future, then our models have potential in accurately measuring the dynamics of the population. As for the interactions between male and females butterflies, our group failed to find any meaningful mathematical relations. For future research, the wasp population can be modeled alongside the butterfly population, making p nonconstant, to yield a more accurate understanding of the fluctuations between these species.

References:

[1] Huigens, M. E., Woelke, J. B., Pashalidou, F. G., Bukovinszky, T., Smid, H. M., & Fatouros, N. E. (2010). Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps. *Behavioral Ecology*, *21*(3), 470–478. doi: 10.1093/beheco/arq007

[2] Cabbage White Butterfly. (n.d.). Retrieved November 9, 2019, from https://www.animalspot.net/cabbage-white-butterfly.html.