SCUDEM IV 2019
Problem C

ES-C-Texas A&M University-Kingsville-Team-4
Ayat Hamide, Stephen Cardenas, Daniel Lerma
Coach: Dongwook Kim
11/9/2019

Simiode Problem C: Chemical Espionage

The purpose of the project was to develop a mathematical model of the interaction of two species as they coexist in an ecosystem. The other objective was to determine what the best balance of the system will be and will happen in the "long run" for the two species of interest.

In the example, the species of interest were butterflies and wasps. Interestingly, the interaction between male and female butterflies also related to the interaction between the wasps and the butterflies themselves. When the male butterflies mate with the female butterflies, they release a chemical signal, called an anti-aphrodisiac. This anti-aphrodisiac covers the females and wards off other male butterflies so that the female can lay her eggs without being bothered by the other males. However, this anti-aphrodisiac also attracts parasitic wasps. If the wasps detect the anti-aphrodisiac, they will follow the female butterfly to the location where she lays her eggs after mating. Then the wasps will lay their own eggs in the butterfly eggs and the wasp larvae will end up eating the butterfly eggs.

The team decided to approach this problem using a predator-prey model, which involves the use of Latka-Volterra equations,

$$dx/dt = \boldsymbol{\alpha}x-\boldsymbol{\beta}xy \qquad (1)$$

$$dy/dt = \boldsymbol{\delta}xy-\boldsymbol{\gamma}y \qquad (2)$$

In these equations, dx/dt and dy/dt are representing the instantaneous growth rates of the two populations with respect to time. Where, alpha is the birth rate of the prey population, and beta is the rate of interaction between the predator and prey species.These equations describe a very basic predator-prey relationship between two species, where many assumptions are made about the environment and other outside factors that may affect the population of either species. However, we can still base our mathematical model on this concept for the specified butterfly and wasp interaction. For our model, we may describe it as,

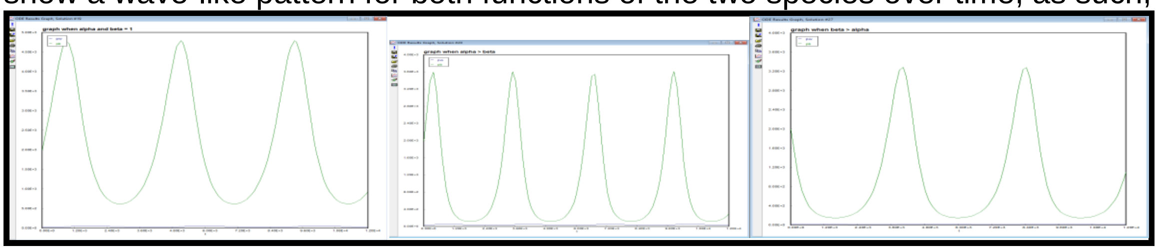
$$dP_b/dt = \alpha k_1 P_b(1-P_b/k_2)-\alpha\beta P_b P_w-k_3 P_b$$

$$dP_w/dt = -k_4 P_w+k_5\, \alpha\beta P_b P_w+k_6 P_w$$

where the table below defines each variable.

	A	B	C	D
1	Variables	Description		
2	P(b)	Butterfly Population		
3	P(w)	Wasp Population		
4				
5	α	Proabability of anti-aphrodisiac increase P(b)		
6	β	Proabability of anti-aphrodisiac attracts wasp		
7	k(1)	P(b) Birth Rate		
8	k(2)	Capacity		
9	k(3)	P(b) Death Rate		
10	k(4)	P(w) Natural Death Rate		
11	k(5)	Conversion Efficiency		
12	k(6)	P(w) Birth Rate		

We know from reasoning, that in a predator-prey relationship, the butterfly population will increase as the wasp population decreases. However, because we are stating that the wasp population is not only dependent on the birth rate, but also relies on k(5), the efficiency of a wasp converting its food, P(b), into offspring, the change in wasp population will begin to increase when the butterfly population is high. Thus, this interaction creates a cyclic pattern of wasps increasing with more food available (increase in butterfly population) and decreasing as they consume too much food and vice-versa with the butterfly population. Representing this on a graph would typically show a wave-like pattern for both functions of the two species over time, as such,



As mentioned before, we have to make a lot of assumptions for this model to make sense. For example, all of the rates are assumed to be constant in the two equations describing the interaction of the species. Not only that, but also, we assume that the butterfly has only one predator and the predator has only one food source and we assume that more of the species do not enter the ecosystem from elsewhere, neither does the species migrate out of the environment they inhabit. Another thing to consider is that the prey has an unlimited food source. With all of this in mind, the model still shows a plausible representation of interaction, when specifically observing two species of a predator-prey relationship.

References

“Lotka-Volterra Equations.” *From Wolfram MathWorld*, mathworld.wolfram.com/Lotka-VolterraEquations.html.

“Mathematical Biology. 14: Predator Prey Model.” YouTube, YouTube, www.youtube.com/watch?v=c2LPt9z4RJY.

“Predator-Prey Model.” YouTube, YouTube, www.youtube.com/watch?v=bRg1cYe-mAA.

“Interactions between Populations.” Khan Academy, Khan Academy, www.khanacademy.org/science/high-school-biology/hs-ecology/hs-ecological-relationships/v/interactions-between-populations.

“Predator-Prey Relationships.” New England Complex Systems Institute, necsi.edu/predator-prey-relationships.

“YouTube.” YouTube, YouTube, www.youtube.com/watch?v=YUjdyKhWt6E.