Executive Summary

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Problem C: Chemical Espionage. This mathematical modeling contest brings us a challenge on applications of mathematics in different disciplines. The cabbage white butterfly *Pieris brassicae* (*P. brassicae*), uses chemical signals to facilitate reproduction. Unfortunately for the butterflies, the chemical signals can be exploited by parasitic wasps. Two species of wasps have been identified that can detect the chemical signature of mated female *P. brassicae*. The wasps tend to follow the signal and lay their own eggs in the butterflies' eggs. These interactions produce two competing pressures on the butterfly population. For the butterflies, the chemical signals enhance reproduction; however, these signals make it more likely their eggs will be eaten by wasp larvae. The goal of this project is to determine the trade-offs and balance between the two competing interests by developing a mathematical model for the interactions of the female and male *P. brassicae* and parasitic wasps.

Assumptions:

1. Sex Ratio of offspring is 1 to 1 for wasps and butterflies
2. Mortality rate for male and female *P. brassicae* is identical
3. Mortality rate for male and female wasps is identical
4. Differences between *T. evanescens* and *T. brassicae* are negligible enough that that they can be combined and averaged for the purposes of their parasitic effects on *P. brassicae*
5. Under field conditions, *T. evanescens* and *T. brassicae* respond to mated *P. Brassicae* in the same way they respond to *P. Rapae* females under lab conditions.
6. Adult mortality rate for *T. evanescens* and *T. brassicae* is 3.45% per day
7. Adult mortality rate for *P. brassicae* is 6.75% per day
8. No other parasites are known to exist in the modeled environment affecting offspring
9. Initial male population is at least 75% of female population in both species
10. *P. brassicae* lay eggs in batches of 50-100.
11. Female wasps lay eggs in batches of approximately 35

Variables, factors, units:

- Bx = *P. brassicae* females, number reaching maturity
- By = *P. brassicae* Male, number reaching maturity
- Wx = Female Wasps, number reaching maturity
- Wy = Male Wasps, number reaching maturity
- t = time, measured in days
- $\underline{\alpha}$ = *P. brassicae* Clutch Size, number of eggs
- $\mathcal{B}$ = Wasp Success Rate, measured as % of mated female wasps
- Δ = Adult mortality rate for *P. brassicae*, measured as % of population
- θ = Wasp Clutch Size, number of eggs
- Ω = Adult mortality rate for wasps, measured as % of population

Model Details:

Although there are various approaches to reproductive models, we chose to represent the reproductive interaction between male and female in the form of $\sqrt{x+y}$ in order to keep our model simple since we are assuming the ratio of male to female offspring will remain one-to-one. We chose to operate the model with two equations, one for the change in butterfly population and another for the change in wasp population:

$$\frac{dB}{dt} = (Bx + By) + \alpha\frac{\sqrt{Bx+By}}{\beta\sqrt{Wx+Wy}}\text{-}\triangle(Bx + By)$$

$$\frac{dW}{dt} = (Wx + Wy) + \beta\theta\frac{\sqrt{Bx+By}}{\sqrt{Wx+Wy}}\text{-}\Omega(Wx + Wy)$$

Where $\underline{\alpha}$, $\mathcal{B}$, $\triangle$, θ, and Ω represent positive constants. $\triangle$ and Ω represent the percentage of adult mortality per day for butterflies and wasps respectively. $\underline{\alpha}$ and θ represent the number of eggs laid per female butterfly and wasp, respectively. $\mathcal{B}$ represents the percentage rate at which parasitic wasps are expected to successfully locate a mated *P. brassicae* female.

The male and female *P. brassicae* and wasps growth rates can be approximated separately using a similar system of equations:

$$\frac{dBx}{dt} = Bx + (\frac{\alpha}{2}\cdot\frac{\sqrt{Bx+By}}{\beta\sqrt{Wx+Wy}})\text{-}\triangle(Bx)$$

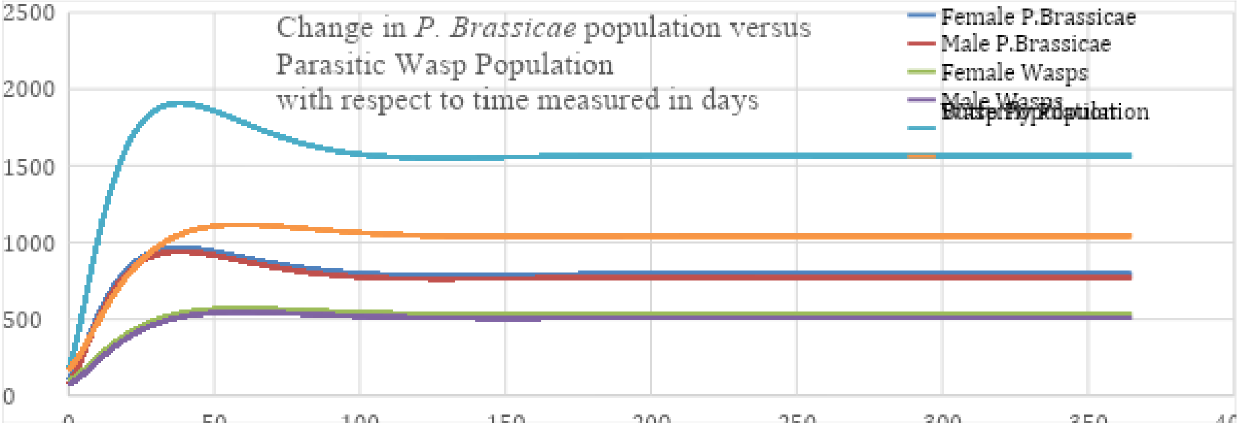
$$\frac{dBy}{dt} = By + (\frac{\alpha}{2}\cdot\frac{\sqrt{Bx+By}}{\beta\sqrt{Wx+Wy}})\text{-}\triangle(By)$$

$$\frac{dWx}{dt} = Wx + (\beta\cdot\frac{\theta}{2}\cdot\frac{\sqrt{Bx+By}}{\sqrt{Wx+Wy}})\text{-}\Omega(Wx)$$

$$\frac{dWy}{dt} = Wy + (\beta\cdot\frac{\theta}{2}\cdot\frac{\sqrt{Bx+By}}{\sqrt{Wx+Wy}})\text{-}\Omega(Wy)$$

However, among the undesirable features of this model is the prediction of an unrealistic rate of population growth when there exists a large excess of the more common sex.

Results/conclusion:
Our model suggests that the two species adjust to an equilibrium state rapidly and tend to maintain equilibrium. Our model produces a reasonable conclusion as long as our assumptions hold. However, if the environment changes to favor the survivability of one sex in either or both species, our model will produce unreasonable growth rates based on the more common sex.



Reflection:

In the end this is a challenging problem; it is both interesting and nuanced. We could not have reached a reasonable solution and prepared the corresponding presentation without the individual strengths of each team member. Overall, the process of breaking down a problem and working with a team to create a mathematical solution was a valuable experience.