SCUDEM: Executive Summary (Coach: Matthew Dobson)

SCUDEM Problem: Problem C, Chemical Espionage

Group members: Frank Cole, Vishvesh Gandhi, Alexander Shilcusky

School: University of Massachusetts, Amherst

Coach: Matthew Dobson

Summary: We attempt to model the relationship between the Pieris Brassicae butterflies and the parasitic wasps through a coupled system inspired by the Lotka Volterra method for modeling predator-prey relationships in nature. The problem at hand is to model the dynamics of a host-parasite relationship, so we had to alter the Lotka-Volterra model significantly. A number of qualitative aspects of the system would be lost by simply treating the butterflies as 'prey' and the wasps as the 'predator. For one, we must consider the interactions between both the male and female populations of the butterflies. When the ratio of males to females changes, the dynamics of their population are affected, as one might guess. We also have to consider the various differences that a host-parasite relationship entails. The parasitic wasps are not eating the butterflies themselves, but rather infiltrating their nests and laying eggs - which, when hatched, will yield larvae that eat the butterfly eggs.

Our derivation of our mathematical model begins at the conceptual level, and we try to describe what we want to see qualitatively in the solutions to the differential equations before describing the system using math language.

Our model depends on five parameters: α, the growth rate of the butterfly population, σ, the natural death rate of the population, β, the rate at which the wasps consume the host's eggs, δ, the growth rate of the parasitic wasps, and γ, the natural death rate of the wasps. We perform our numerical calculations entirely in Python. We use a Runge-Kutta method to solve the differential equation, and include plots of the time evolution of both population, as well as phase portraits which plot a competing population on each axis and display the trajectories of solutions.

We consider how changing each of the parameters would affect the long term dynamics of the system. We also show what happens with different initial populations for the host and parasite, including the impact of having a different initial ratio of male to female butterflies. To conclude, we present an optimal choice of parameters and initial conditions that yield complete stability for both the host and parasite populations in all time! For each piece of numerical data we tried to provide apt justification, both mathematically and biologically.