<u>Executive Summary Prompt C Team 1 UMBC</u>

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Statement of Problem: Model interactions between male and female *P. Brassicae* butterflies and *T. Evanescens* and *T. Brassicae* wasps.

Our system is defined as the best balance for *P. Brassicae* butterfly survival rate. The males of the *P. Brassicae* butterflies compete against one another to fertilize the female butterfly. The spermatophores contain an anti-aphrodisiac, benzyl cyanide (BC), the levels of which are directly proportional to the level of proteins the male was able to provide the females. The higher the levels of BC, the stronger the male competitor. The anti-aphrodisiac is emitted by the female to dissuade other male suitors. Here, we assume that all of the BC given by the male is emitted by the female. However, the two species of wasps, *T. Evanescens* and *T. Brassicae*, use the blended chemical odor to find these mated female butterflies and parasitize her nest.

The wasps can detect BC at 2 µg, and are attracted until BC levels are 20 µg, at which point they are repelled. This interval represents the area at which the butterflies are most vulnerable. To prevent detection by wasps, it is favorable for female butterflies to emit less than 2µg of BC. However to favor sexual selection, the male butterfly still wishes to maximize his BC content, and therefore will try to optimize for a BC level that is as close to 2µg as he can get. This selection competition is represented through this governing equation:

$$C_{BC,\,net} = Wasp\ Threshold\ -\ C_{BC,MAX} \tag{1}$$

Where, $C_{BC,\,net}$ is the minimized difference between the wasp threshold and $C_{BC,MAX}$. $C_{BC,MAX}$ is the maximized C_{BC} that does not break the wasp threshold

T. Evanescens has a learned response to the chemical signal, therefore only after a successful parasitization does this wasp know to continue searching for this chemical blend. However, *T. Brassicae* has an innate response to the chemical blend, and therefore will always seek out the mated female butterflies. If modeling the effects of the *T. Brassicae* wasp, we can assume that with respect to the BC levels alone, there is a constant selective pressure on the butterflies to decrease their anti-aphrodisiac output. Because the effects of the *T. Evanescens* wasp are dependent on the success of both mounting and successful oviposition in previous parasitizations, there is a 25% chance of the wasp offering selective pressure on the butterflies. Since we are looking at a mixed population, for the sake of simplification we are assuming constant selective pressure.

The male and female interactions are an integral part of the system dynamics. Because the wasps depend on the BC levels emitted by the female, who receives the compound from the male, we decided to model the level of BC per mated female butterfly as a function of the degradation of the male spermatophore in the female bursa sac.

This degradation function is defined through a simplified version of Fick's Second Law of Diffusion:

$$\frac{\partial C_{BC}}{\partial t} = D_{AB}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_{BC}}{\partial r}\right)\right) \tag{2}$$

We used a spherical approximation of the spermatophore. The degradation happens solely in the radial direction, therefore we are able to eliminate the θ and ϕ related terms. The term D_{AB} was closely approximated to the diffusivity constant of proline in an aqueous solution. This was because the protein composition of the indigestible outer envelope is rich in proline. However, through modeling and the constraints of our boundary conditions, we were able to iteratively find a better D_{AB} . We also assumed a 20 µL volume, as that is the volume used in our reference article, and allows control over the concentration combinations available.

Our model was graphed in MATLAB, using the pdepe function solver. We used established spermatophore radii ranges from literature. The time limit was based off of the average amount of time the female takes to degrade a single spermatophore- 30 hours, or 108,000 seconds. The recovery time and reinitiation of mating have not been accounted for, as the model focuses on a single generation.

The model produces curves similar to the middle curve in the figure to the right. The two optimization options that we hold are either allowing the $C_{BC,MAX}$ to be greater than the repellant threshold (20 µg), or to have it below the detection threshold (2 µg). If we follow the curve going above the repellant threshold, we still have a time interval in which the wasps can detect the mated female butterflies. However, by decreasing the $C_{BC,MAX}$ to be below the detection threshold we allow for complete butterfly survival rate for that generation. This optimized curve is our best balance for the system.