Problem C: Chemical Espionage of *P. brassicae* and *T. brassicae*

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Background:

Pieris brassicae, or the large white cabbage butterfly, is one of the most commonly found species of butterflies all over the world. When these butterflies mate, the male butterfly sprays the female with Benzyl Cyanide, which is an anti-aphrodisiac. Spraying this chemical deters other male butterflies from bothering impregnated female butterflies. In turn, the females are able to search for a more optimum location to lay their eggs. However, a species of parasitic wasps, *T. brassicae*, has learned to take advantage of the anti-aphrodisiac spray. When in close range with one another, the wasps detect the Benzyl Cyanide on mated female butterflies and latch on to the butterflies. The wasps ride the cabbage butterflies back to their eggs, and parasite the unborn caterpillars. A trade-off emerges from this situation, as the anti-aphrodisiac allows the females to find the best spot to lay their eggs but exposes them to potential parasites. Using differential equations, a system of equations was created to model the relationship between the population of butterflies and the population of parasitic wasps. The goal was to determine the ideal percentage that butterflies should spray the anti-aphrodisiac, in order to optimize the population of butterflies.

Significant Factors:

The first step in creating the system of equations was to determine significant factors that would impact both the butterfly and wasp population. The first factor was the number of successfully hatched butterfly eggs per year, or the birth factor (B_F). The birth factor was characterized by the current female butterfly population, number egg clusters laid per year, and the number of eggs per cluster. This factor was the main increase in butterfly population in the model. The second major factor was the spray factor (S_f). The spray factor was the independent variable throughout the model and was represented by changing the percentage the male butterflies will spray the females. The encounter factor (E_f) displayed the rate at which the wasps and butterflies would come in contact with one another in nature. To find the chance of encounter, a predetermined encounter area was set from literature [3], and then the random probability that both insects would be found in that same area was determined by multiplying the population density of the butterflies and wasps for the total control area for the entire environment. That probability was then multiplied by the population of the wasps and butterflies. Next was the kill factor (K_f). The kill factor was the percentage of successful parasitisms by the wasps when the insects are found within the designated encounter area. The factor was determined by multiplying the percentage of wasps that hop on to the butterflies and the percent of wasps that hopped on, that then successfully parasitize the eggs [2]. It was assumed that if a wasp was able to make it to the butterfly's eggs, it would kill all the eggs located in that cluster. The next factor the investigated was fecundity (Fe_f). After female butterflies have been sprayed with the anti-aphrodisiac, they normally do not mate again, as other males choose to stay away from that particular female. Fecundity observes the change in butterfly population due to an increase in mating from butterflies now being able to mate more than one time if they were not sprayed with the anti-aphrodisiac [4]. The survivability (Su_v) simply represents the increase in butterfly population over time due to spraying the anti-aphrodisiac. Survivability was used to model how the butterflies are able to find the most optimum egg-laying location from being sprayed. The number of wasp births was also represented in the E_W term. It represents the number of eggs that the wasps lay per cluster of butterfly eggs, leading to new wasps being born. The only main factors left were the natural death of both the butterflies and wasps. These were included by subtracting the current population of butterflies/wasps divided by the average lifespan of butterflies/wasps (L_{BF} and L_W respectively).

Model:
The significant factors described above were used to develop two ODE's that described the change in butterfly and wasp population over time.

$$\frac{dP_{BF}}{dt} = B_R P_{BF}\left[-K_f E_f S_f + S_f\left(Su_v - Fe_f\right)\right] - \frac{P_{BF}}{L_{BF}}$$
$$\frac{dP_W}{dt} = B_R P_{BF} K_f E_f S_f E_w - \frac{P_W}{L_W}$$

The overarching goal of this model is to determine whether it is beneficial for the male butterfly to spray the female butterfly with the anti-aphrodisiac compound. The variable being optimized is the spray factor, as this has a critical effect on the increase or decrease in population of the wasps and butterflies The spray factor, along with the population of butterflies and wasps, were written in terms of deviation variables. After the ODE's were written in terms of deviation variables, a step change in the spray factor was introduced to observe the deviation from the steady-state value of population. The differential equations were solved numerically and then plotted as population of butterfly and wasps over time. Figures 1 and 2 show how the population of the wasps and butterflies change over time when a step change in the spray rate of male butterflies is introduced.

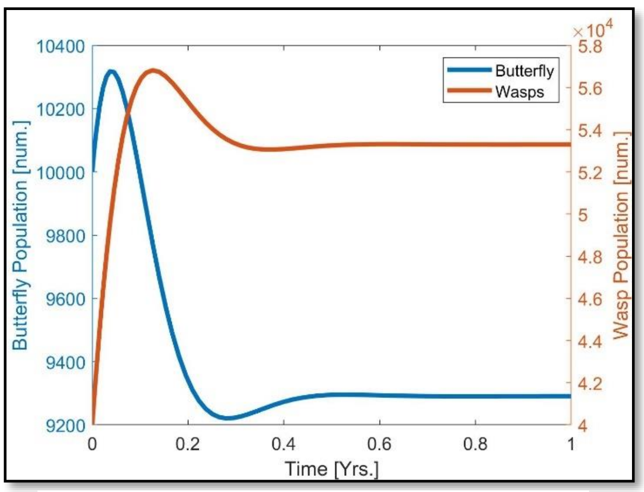

Figure 1: 50% increase in Spray Rate

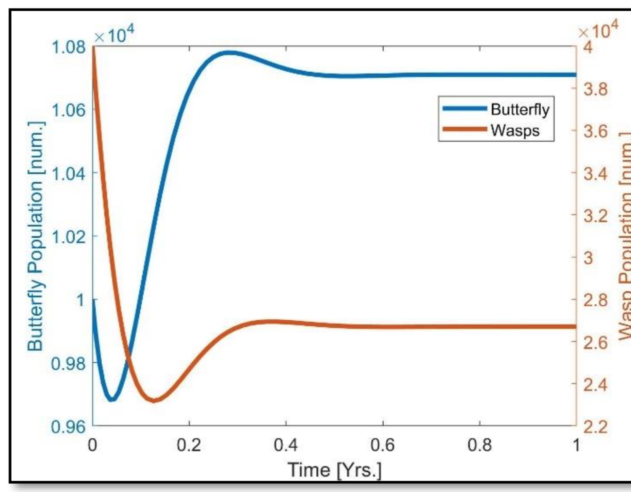

Figure 2: 50% decrease in Spray Rate

Conclusion:
In figure 1, the butterfly population spikes at the beginning, as the survivability takes hold and the butterflies are able to find great locations to lay their eggs. However, shortly after, the kill factor becomes the dominant driving force, and the wasps begin to encounter more and more butterflies treated with the anti-aphrodisiac. In figure 2, the opposite happens. The survivability becomes far more important, and the butterfly population outweigh the wasp population. To maximize their population, the butterflies should spray as little as possible.

References:

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