SCUDEM 2019: Flight of the Butterflies, Problem 3

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1 Introduction

The butterfly species *Pieris brassicae* utilizes a form of chemical signaling to find mates. The female butterfly will release pheromones that attract male butterflies. When the male butterfly finds a female, it will release its own pheromones that repel other males, thus reducing competition to mate with the female. However, the pheromone released by the male has the side effect of attracting species of parasitic wasps including *Trichogramma brassicae* and *Trichogramma evanescens* [2]. The parasitic wasps will then hitch a ride on the female butterfly and lay its eggs in the butterfly eggs.

2 Model Assumptions

We make the following assumptions:

- The ratio of born male to female butterflies is 3:2 [1]
- Butterfly eggs are destroyed only by wasp larvae
- A certain proportion of butterflies and wasps die every day due to natural causes, wasp larvae and butterfly eggs do not, they are safe in their nests.
- Wasp larvae can only be laid in butterfly eggs
- The insects do not have a breeding season - mating behavior is invariant in time
- Populations are evenly distributed in space

3 Model

We first find basic data about the *Trichogramma brassicae* and *Pieris brassicae*. For our model we say that the entire system is defined by 5 populations: The butterfly eggs: E, the male butterfly population: M, the female butterfly population: F, the wasp larvae population: L, and the adult wasp population: W. We first define the lifetimes of each of the different populations, labeled μ_i where i is the population. Let $\mu_M = \mu_F = 23$ days, $\mu_L = 7$ days, $\mu_W = 10$ days, and $\mu_E = 50$ days [3, 4]. While the lifetime of eggs is actually 12 days, we can account for the pupal and larval stage of the butterfly in the egg stage, as we assume that there is no extra interactions in those stages. We then find the maximum amount of eggs and wasp larvae that the given populations could produce in the next day. We argue that the function is essentially a limiting reagent problem; if the male population is much larger than the female population, the birthrate will be limited by the female population, and vice versa. To address this, we introduced minimum functions seen below.

$$B_0(t) = \min\left(c_M M(t), c_F F(t)\right) \qquad W_0(t) = \min\left(\xi c_M M(t),\ \xi c_F F(t),\ c_W W(t)\right) \tag{1}$$

Here, $c_m = 25$ is the chosen to represent the max amount of butterfly eggs a single male can fertilize in one day, therefore $c_m M(t)$ should be the max amount of eggs the entire male population could contribute, and likewise for the female population. The minimum of these functions, B_0, then represents the max amount of eggs the butterfly population could produce in the next day. Likewise, the minimum of B_0 and the wasp population is C_0 and will be the maximum amount of wasps that can be produced in one day. We choose, $c_F = 5$ eggs per day and $c_W = 15$ larvae per day. Given these constants we create differential equations for each population

$$\frac{dE}{dt} = \alpha[B_0(t) - B_0(t - \mu_E)] - \beta\xi W_0(t - \mu_E) \tag{2}$$

$$\frac{dB}{dt} = \frac{5}{3}\frac{dM(t)}{dt} = \frac{5}{2}\frac{dF(t)}{dt} \tag{3}$$

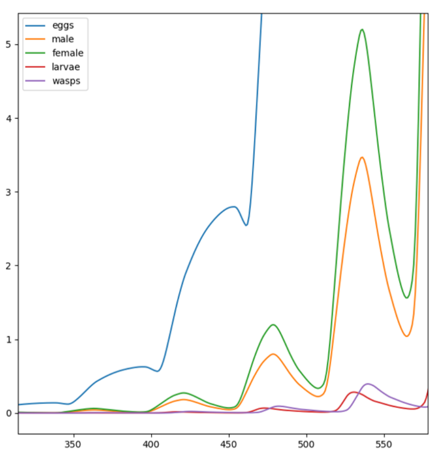
$$\frac{dM}{dt} = \frac{\alpha}{3}[B_0(t-\mu_E) - B_0(t-\lambda_B)] - \frac{\beta\xi}{3}[W_0(t-\mu_E) - W_0(t-\lambda_B)] + \ell_{\mu_B}[\alpha B_0(t-\lambda_B)] - \delta M(t) \quad (4)$$

$$\frac{dL}{dt} = \beta[W_0(t) - W_0(t-\mu_L)] \quad (5)$$

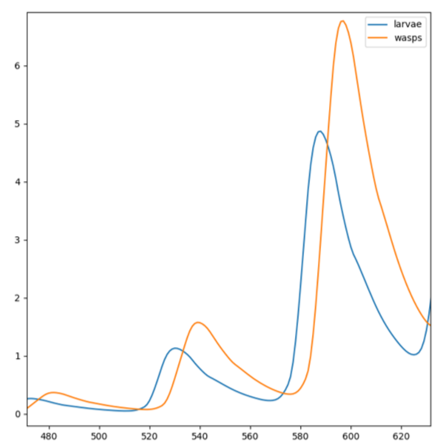
$$\frac{dW}{dt} = \beta[W_0(t-\mu_L) - W_0(t-\lambda_W)] + \ell_{\mu_W}[W_0(t-\lambda_W)] - \delta W(t) \quad (6)$$

We use the terms α and β to denote the strenth of the aphrodisiac and anti-aphrodisiac respectively. We model α to suppress the growth rate of the eggs using $\alpha = exp(cE(t)/F(t))$. In the Wasp and Butterfly population we have two terms yet un-discussed, $\ell_{\mu_B}[\alpha B_0(t-\lambda_B)$ and $\delta M(t)$. In this model δ is the percentage of the population that dies due to natural causes every day. This is an inclusive term that accounts for all random events that can lead to an animals death. However, since this term was introduced, we had to make sure that the butterflies that die due to old age were not the entire population of butterfly that was born a butterflies lifetime ago. Since every day a certain proportion of butterflies died, the total number of butterflies dying of old age at time t is equal to the population of butterflies born a lifetime ago minus the number of butterflies that died of natural causes in that generation. This value is calculated in the ℓ term and is equal to: $\ell = (1-\delta)^{\mu-1}\delta P_0$ where P_0 is the initial population. To numerically solve these models we used Euler's Method with step sizes of one day.

4 Results



(a) A subfigure



(b) A subfigure

Our model failed to produce any stable results. We observe that all populations die when α goes to 0. Due to time constraints, we were unable to implement a food scarcity functionality in our model, and as a result, the populations grow exponentially and indefinitely. The way our model was created made adding additional functionality or constraints on our system complicated and exacting. In order to include a food scarcity term we would have to add the term to the egg population, a correcting term in the egg population, and then carry that term through to the butterfly population to properly account for the maturing eggs. While we originally thought that constructing a model in this way would allow for more accurate computations in regard to non-uniform populations, we found that it was difficult to adapt and easy to make mistakes that led to unreasonable outcomes. We believe that this problem can be better modeled using an agent-based approach.

5 References

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