Problem C - Chemical Espionage

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The Large Cabbage White Male Butterfly produces a species specific anti-aphrodisiac during mating that adheres to the female and repels other males from approaching the female to mate. While the anti-aphrodisiac benefits the female by allowing her time to deposit her eggs in preferable locations, it also can prove detrimental to her offspring. Two parasitic wasp species are known to exploit these chemicals and, unbeknownst to the butterflies, lay their wasp eggs inside of the butterfly eggs (one species does this innately, the other learns to); in the egg the wasps consume the butterflies [1]. Our team investigated the relationship between pheromone concentration and species survival, as well as constructing a model that represent the both species population given a time.

The Lotka-Volterra equations (eq. 1) are used to represent a predator/prey system.

$$\begin{aligned} \frac{dB}{dt} &= \alpha B(t) - \beta B(t)W(t) \\ \frac{dW}{dt} &= \delta B(t)W(t) + \gamma W(t) \end{aligned} \tag{1}$$

For our model, $\frac{dB}{dt}$ and $\frac{dW}{dt}$ represent the population growth rates of the butterflies and wasps respectively. In addition, α represents the butterfly birth rate, β and δ represents the rate at which the butterflies and wasps meet, and γ represents the wasps' decrease rate.

For simplicity, we are assuming there is only one species of wasp, and that this species has the innate ability to take advantage of the anti-aphrodisiac pheromone. Future work on this model would include adding a second wasp species that *learns* their chemical espionage after one successful parasitism. We also assume that the amount of Male and Female butterflies are half of the butterfly population.

Our estimate of α is the birth rate (eggs/time) times 0.5 (only females produce eggs) times the survival rate (k_1*pheromone) because a higher anti-aphrodisiac pheromone yields better egg placement, i.e. better survival rate. In addition, if there is no pheromone, then we are assuming the butterflies are not mating, and will die out, thus we add a death rate constant to $alpha$. The female oviposits 2-3 days after mating, and will oviposit for 8 days, yielding 10-11 days between mating. However a female can mate 5 days after initially mating [2]. We are taking the average number of days for 1 brood to be uniformly distributed between 5 and 11: 8. Similarly, since a butterfly lays 20 to 50 eggs per brood [1], we are estimating the number of eggs per brood to be 35. Thus our estimated birthrate is 35/8. Note that if we had collected ample sample data, since it's reasonable to believe in reality number of eggs laid per female butterfly follow Poisson distribution, we would ideally like to estimate the true rate λ by finding the sample mean. Finally our death rate for the butterflies is given as simply 1/44.5 because Bhowmik and Gupta state the life cycle of P brassicae lasts an average of 44.5 days [3].

Both β and δ are related to the rate at which the wasps and butterflies meet, which is directly proportional to the pheromone concentration times some constant k_2. Here we assume that the wasp will successfully parasitize every egg clutch it comes across. Our model sets β and δ to be equal. For future work, we may consider varying them by a constant.

For the last variable γ, we are only considering the death rate in this model. In future improvements of the model, we would want to look into a more representative γ. However, for now, assuming food sources once grown are limitless, this suffices.

Our approximation of the butterfly wasp population system will be as follows:

$$\begin{aligned} \frac{dB}{dt} &= \frac{35}{8}\frac{1}{2}k_1 pB(t) - \frac{1}{2}k_2 pB(t)W(t) \\ \frac{dW}{dt} &= \frac{1}{2}k_2 pB(t)W(t) + \frac{1}{17}W(t) \end{aligned} \tag{2}$$

For gathering our results, we set $k_1 = 10^4$, and $k_2 = 10^2$. These constants represent how much weight the anti-aphrodisiac carries for the survival of the butterfly eggs based on their placement, and how much the anti-aphrodisiac attracts the wasps. We have chosen $k1 > k2$ because for the use of the pheromone to benefit the butterflies, the pros should, for a reasonable amount of pheromone and initial conditions, out weight the cons. We changed the values of the k's while keeping the initial conditions and pheromone constant. We observed that the overall behavior in the long run of both species did not change for the most part when we increased or decreased either constant, however the values of $W(t)$ and $B(t)$ changed. As a result, our next steps would be to better estimate k_1, and k_2.

Nevertheless, keeping k_1, and k_2 constant and setting $B(0) = 200$ and $W(0) = 100$ we varied the pheromone constant to observe the results. We found that for a very low amount of anti-aphrodisiac, our model shows that the wasp population will die out and butterfly population will soon follow. As expected, with low pheromone the wasp population will not be able to detect the butterfly and will die. Our model shows the butterfly population decreasing to 0 because the pheromone level is not high enough to deter male butterflies and the females will not be able to oviposit in safe locations. We also found that for a large range of concentration of pheromone both populations will oscillate but neither will go extinct. This is the balance of pheromone that is enough to keep both populations alive. Finally, when too much pheromone is in the system we found that the wasp population will drastically increase while the butterfly population goes to 0 almost immediately, and then the wasp population follows.

There are many ways we can improve our model. One of them is to look at our pheromone constant. In the future we would like to consider p^3 instead of p because concentration should decrease by $distance^3$. We would also like to introduce the second wasp species and play around with the initial conditions. This will create more competition for the wasp species we are currently modeling. We would also like to better understand the sensitivity of our model based on changes in our coefficients. Finally, we think it would be beneficial for the model to better capture the life cycle of butterflies and wasps. Both species go through multiple changes in their life cycle and accounting for these differences may affect the outcome of the system.

Overall, we believe our model captures the general relationship between the Parasitic Wasp and the Large Cabbage White Butterfly. For a large range of concentrations, while population fluctuations differ, the populations do not die out. The model depicts many interesting scenarios depending on pheromone and initial conditions, and presents us with a lot of scenarios to interpret.

References

[1] Martinus E. Huigens et. al. Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking trichogramma wasps. *Behavioral Ecology*, 21(3):470–478, 2010.

[2] W.A.L David and B.O.C Gardiner. Oviposition and the hatching of the eggs of pieris brassicae (l.) in a laboratory culture. *Bulletin of Entimological Research*, 53(1):91–109, 1962.

[3] M. Bhowmik and M.K. Gupta. Biology of cabbage butterfly pieris brassicae linn. (lepidoptera: Pieridae). *International Journal of Current Microbiology and Applied Sciences*, 6(12):3639–3644, 2017.