Problem: C – Chemical Espionage
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Background: The females and males of the butterfly species *Pieris brassicae* employ chemical signals to increase their chances of breeding (Huigens *et al.*, 2008). The females attract all viable mates with an aphrodisiac, but the males counteract this effect after breeding with a female by producing an anti-aphrodisiac that dissuades other males from breeding with that female. The decreased male attention is advantageous for both sexes, as the male will sire more of that female's offspring and the female will not be accosted by other males and will therefore have more time to select ideal sites for her eggs. However, female parasitic wasps can detect the anti-aphrodisiacs and will hitchhike on the mated females to parasitize her freshly laid eggs. In this model, we show how both sexes of *P. brassicae* and the parasitic wasps interact.

Assumptions: We assume that there are regular (produce anti-aphrodisiac) and null (do not produce anti-aphrodisiac) males in the population. The inheritance pattern of the anti-aphrodisiac trait is also y-linked (found on the Y chromosome) and controlled by one gene. Both species have a 1:1 sex ratio and neither species experiences significant predation or starvation. In this model, all parasitized butterfly eggs will not hatch.

Model: To model this interaction, we used the Lotka-Volterra predator-prey equations as a base to reflect the relationship between the *P. brassicae* eggs and the parasitic wasps (Bacaër 2011). Additionally, we used the Red Queen theory to relate how the favorability of the regular and null phenotypes changes as the parasitic wasp population fluxes (Morran *et al.*, 2011). Table 1 contains the variable definitions and values and equations (1) through (4) are the differential equations used to model this scenario. While some variables were generated based upon other resources, others were documented in other studies, and the initial population values were chosen at random (Huigens *et al.*, 2008; Aslam *et al.*, 2000).

Variables Definition	**Variable**	**Value**
Population of butterflies	P_N	P_0 = 1000
Population of non anti-aphrodisiac (NA) male butterflies	M_{NA}	M_{NA} = 475
Population of non anti-aphrodisiac NA male butterflies	M_{AA}	M_{AA} = 25
Population of wasps	w_N	w_0 = 50
Brood size	b	141
Survival to adulthood of butterfly population	P_s	0.42
Survival ratio of offspring of NA male butterflies	S_{NA}	0.65
Survival ratio of offspring of anti-aphrodisiac AA male butterflies	S_{AA}	0.95

Chance of parasitic wasp finding eggs if female mates with NA male	i_{NA}	0.01
Chance of parasitic wasp finding eggs if female mates with AA male	i_{AA}	0.025
Death rate of butterfly population	d_p	0.5
Death rate of wasp population	d_w	0.7

Table 1. Definitions and values of variables used in modeling equations.

$dP_N/dt = [(dP_{AA})/(dt)+(dP_{NA})/(dt)]$ (1)
$dP_{NA}/dt = S_p(S_{NA}*M_{NA(N-1)}*b[1-w*i_{NA}])- dP_{NA(N-1)}$ (2)
$dP_{AA}/dt = S_p(S_{AA}*M_{AA(N-1)}*b[1-w*i_{AA}])- dP_{AA(N-1)}$ (3)
$dM_{NA}/dt = (dP_{NA}/dt)*(½)$ (4)
$dM_{AA}/dt = (dP_{AA}/dt)*(½)$ (5)
$dw/dt = w_{(N-1)}*b*[(i_{NA}*M_{NA(N-1)})+(i_{AA}+M_{AA(N-1)})]-d_w*w_{(N-1)}$ (6)

Reflection: The ideal state for the *P. brassicae* population is to maintain a small pool of non anti-aphrodisiac males that can become more prevalent and counteract slowing population growth due to rising numbers of parasitic wasps. This would prevent large population crashes for both species that the Lotka-Volterra equations tend to predict.
Improvements: To refine the model, collecting new data concerning the inheritance mechanism, the effect of environmental factors on both populations, and the actual population numbers across generations would be useful. Existing data concerning how female butterflies preferentially lay eggs on young plants of specific species could be incorporated to alter the likelihood of egg parasitization, given the chance a hitch-hiked female could carry a wasp with her.

References:
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