SCUDEM Executive Summary:
Problem C

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Stability of *Pieris brassicae* butterfly and parasitic *Trichogramma brassicae* and *Trichogramma evanescens* wasps

Problem statement

We modeled the population trends of the butterfly *Pieris brassicae*, who host the parasitic wasps *Trichogramma brassicae* and *Trichogramma evanescens*. The male *P. brassicae*, after mating, emit an anti-aphrodisiac to protect their mate from the harassment of other males. This anti-aphrodisiac has the unintended effect of attracting the parasitic wasps, who use the chemical signal to identify and mount female *P. brassicae*. The wasps lay their eggs inside the *P. brassicae* eggs, and upon hatching, the wasps consume the butterfly eggs, killing the butterflies.

We modeled the population trends of an ecological system where *P. brassicae*, *T. brassicae*, and *T. evanescens* are the three primary species. From this, we determined how the initial populations within the community affect the long-term stability of such an ecological system.

Major Assumptions

1: *T. brassicae* and *T. evanescens* can parasitize the same clutches.

Justification: The proportion of clutches parasitized by each wasp species is so low that the proportion of clutches parasitized by both is minimal (about 0.06). On average, this will affect the population of either wasp by less than one percent. Additionally, there was no literature showing explicit interactions between the two wasp species within a single clutch.

2. All three species have the same rate of death.

Justification: This assumption was made for simplicity due to lack of specific data about the wasp species in literature. This justification is reasonable because all three species share the same habitat, have similar generic life cycles, and are all insects.

Model

Our model considers 5 distinct populations: male *P. brassicae*, unmated female *P. brassicae*, mated female *P. brassicae*, *T. brassicae*, and *T. evanescens.*

Term	Definition
M	Male *P. brassicae* population
F_u	Unmated female *P. brassicae* population
F_m	Mated female *P. brassicae* population
TB	*T. brassicae* population
TE	*T. evanescens* population

Equations:

We define the following system of differential equations to describe mating seasons:

$$\frac{dM}{dt} = (Male\ P.B.\ hatchlings) - (Death\ constant) * M$$
$$\frac{dF_u}{dt} = (Female\ P.B.\ hatchlings) - (Death\ constant) * F_u - (Rate\ at\ which\ females\ mate)$$
$$\frac{dF_m}{dt} = (Rate\ at\ which\ females\ mate) - (Death\ constant) * F_m$$
$$\frac{dTB}{dt} = (T.B.\ hatchlings) - (Death\ constant) * TB$$
$$\frac{dTE}{dt} = (T.E.\ hatchlings) - (Death\ constant) * TE$$

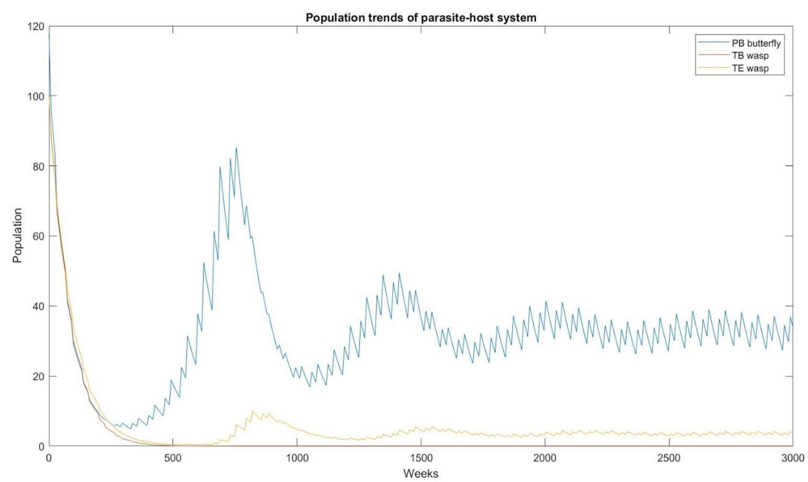
The number of *P. brassicae* hatchlings and the number of wasp hatchlings are positively related to the number of mated females. The number of *P. brassicae* hatchlings and the number of wasp hatchlings are negatively related because all hatchlings require the same resource, the *P. brassicae* egg.

The rate at which females mate is proportional to $\frac{M*F_u}{F_m}$. The numerator expresses that mating requires interaction between males and females. The numerator involves the number of mated females because mated couples release anti-aphrodisiacs that deter nearby males from mating.

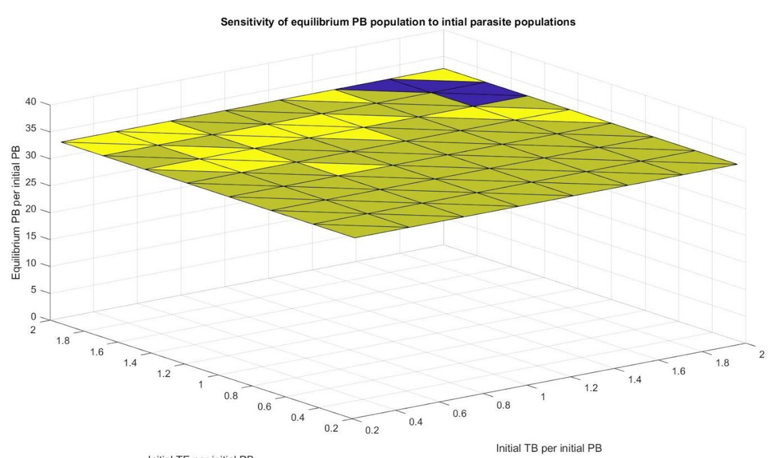
Between mating seasons, the number of *P. brassicae* hatchlings is reduced to zero. Wasp hatchlings are also reduced to zero because the wasps rely on *P. brassicae* mating behavior for reproduction. The death constant is adjusted during this period because the organisms remain as eggs for a long duration (contrasting the large quantity of adult organisms during mating seasons).

Results:

Using MATLAB, we find that the ecological system will be stable for almost all initial conditions. Below is the profile of the system with initial populations of male *P. brassicae*, unmated female *P. brassicae*, mated female *P. brassicae*, *T. brassicae*, and *T. evanescens* equal to 63, 45, 5, 100, and 100, respectively.



By evaluating the system's sensitivity to initial conditions, we find that the total population of *P. brassicae, T. brassicae,* and *T. evanescens* will always converge to 32.74%, 0.00%, and 3.54% of the initial *P .brassicae* population, respectively, as long as all initial populations are positive.



Conclusion:

An ecological system with primary species *P. brassicae*, *T. brassicae*, and *T. evanescens* will reach long-term stability for all sets of initial conditions.