Problem B: Movement of an Object in Microgravity Environments

SCUDEM Competition

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Abstract

Asteroids are rocky bodies of varying shapes that orbit the sun [1]. Some asteroids are made of clay or metals, such as, platinum, iron, and nickel [1]. Also, asteroids could be as small as pebbles or as large as a planet. These asteroids were remnants from the formation of the solar system [1]. The study of these rocky bodies are very essential because they contain vital information about the planets and the sun, since the asteroids were created simultaneously as the planets and sun were formed.

One instrument that is used to study asteroids is a probe. The probes used in studying asteroids are sophisticated spacecrafts that contain sensors [5]. These probes collect scientific information that are further studied by scientists [5].

There are several challenges that are encountered when collecting these scientific information using a probe. For example, the habitat of the asteroid is a low gravity environment of 10^{-6} [4]. Thus, the probe has to have a coordinated motion in this environment. The main goal of scientists, engineers, and mathematicians is to utilise different known laws in order to determine a simplified or complex model that can be used to proffer solutions to these challenges.

For this project, we intend on landing our small probe on a small asteroid. We only considered the vertical descent of the probe once it is at a short distance, r, from the asteroid. We have also assumed that the air resistance is negligible [2].

To create a model, we have also named several parameters which are: F_G, the gravitational force of the asteroid; G, the gravitational acceleration of the asteroid; a, the acceleration of the probe; m, the mass of the probe; M, the mass of the asteroid; h, the step function; and v, velocity of the probe.

When the probe is launched into space and it approaches the targeted asteroid, we formulated a model called the landing model that will be

used to land the probe, as close as possible, to a predetermined point. This equation is modelled after the Newton's second law of motion, which states that the motion of a body is determined by the sum of the forces acting on the body [3]. The following resultant forces are given as:

$$x''(t) = \frac{-Gmx}{(x^2+y^2)^{\frac{3}{2}}} \ \& \ y''(t) = \frac{-Gmy}{(x^2+y^2)^{\frac{3}{2}}}$$

The initial values that are used to solve the differential equations are represented in the following equations:

$$t_{i+1} = t_i + h$$

$$v_{x_{i+1}} = v_{x_i}$$

$$r_{i+1} = \sqrt{x_{i+1}^2 + y_{i+1}^2}$$

As the probe lands on the asteroid, we decided that we would like to collect different samples from different locations on the probe. Thus, the probe would have to move (hop) around the asteroid. This motion was not assumed to be generated by thrust − a force or push generated by the probe that moves it in the opposite direction (away from the asteroid). We assumed that the probe was equipped with a spring to aid in this hopping motion. We derived that the minimum amount of energy that is required to move the probe is the sum of the kinetic energy of the probe and the spring potential energy, given by Hooke's Law. This is minimal energy, E, is given as:

$$E = \frac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

Furthermore, the bounce model was used to determine the amount of bouncing that the probe should undergo to prevent damaging the asteroid. The bounce model is assumed to follow a geometric series. This model relates the total distance,X, travelled by the probe to the height of each bounce made by the probe, h. The resulting equation is:

$$X = \frac{2h}{1-f}, \text{ where } f \text{ is the fraction of the previous height}$$

References

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