SCUDEM 2019

Western New England University Team

Team Members

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Coach

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Chosen Problem

Problem C: Chemical Espionage

Executive Summary

Problem C laid out a situation which on the surface seemed like the interaction between a predator and its prey. Specifically there is this population of butterflies in which the males will release an anti-aphrodisiac to stop other males. This seems typical of most species; eliminating the competition makes the path to victory easier. However, the chemical that is released has a downside, it makes it easier for wasps to find the butterfly eggs and plant their larvae inside of them, devouring the eggs. One would expect that there is some balance to be reached between the two populations and the concentration of aphrodisiac.

The model used was derived starting from the Lotka-Volterra predator-prey model. This seemed like a good starting point since the butterflies could act as the prey and the wasp larvae could act as the predators. Furthermore, the equations do not model the population of the butterflies, it instead models the population of butterfly eggs. After all, the wasps are attacking only butterfly eggs. Another motivator was if one knew how the butterfly eggs were changing they could infer the change in the butterfly population.

The question of how to put the aphrodisiac into the equations was a difficult part of the modeling. The amount of aphrodisiac was assumed to be proportional to the current amount of eggs. The logic was, if the current amount of eggs were high then there must be a high amount of aphrodisiac because a large amount of males must have mated to create the large amount of eggs. Moreover, the amount of aphrodisiac affects the growth rate because as stated in the problem, it positively affects the population of both the larvae and the butterfly. Therefore the aphrodisiac should be associated with the positive terms of each equation.

While analyzing the model the solutions had extreme volatility. The model was adjusted by squaring the negative predator term in each equation. This gave possible steady state solutions with less sensitivity for a range of initial conditions. For the initial condition of the wasps ranging from 1-20 there is very little sensitivity and that variance gives no drastic changes. However, if the initial wasp population is either greater than 20 or greater than the initial amount of butterfly eggs, the results of the equations became very skewed. Also, the set of equations becomes volatile if the initial amount of eggs in the system is set at a very high value, over 2000-3000.

x(t) = population of butterflies (in hundreds) t days after the start,

y(t) = population of wasps (in tens) t days after the start,

a = relation to anti-aphrodisiacs

b = rate of wasp larvae layed

d = death rate of wasps

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Clear[x, y, xn, yn, apprx, a, b, c, d];
a = .1;
b = .2;
d = .2;
eqs = {x'[t] == a * x[t] - b * x[t] * y[t]^2, y'[t] == -d * y[t]^2 + a * x[t] * y[t], x[0] == 100, y[0] == 5};
apprx = NDSolve[eqs, {x, y}, {t, 0, 365}];
xn = x /. apprx[[1]];
yn = y /. apprx[[1]];
Plot[{xn[t], yn[t]}, {t, 0, 365}, PlotStyle → {Blue, Red}, PlotRange → {0, 5}]
Plot[{xn[t], yn[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}, PlotRange → {0, 105}]
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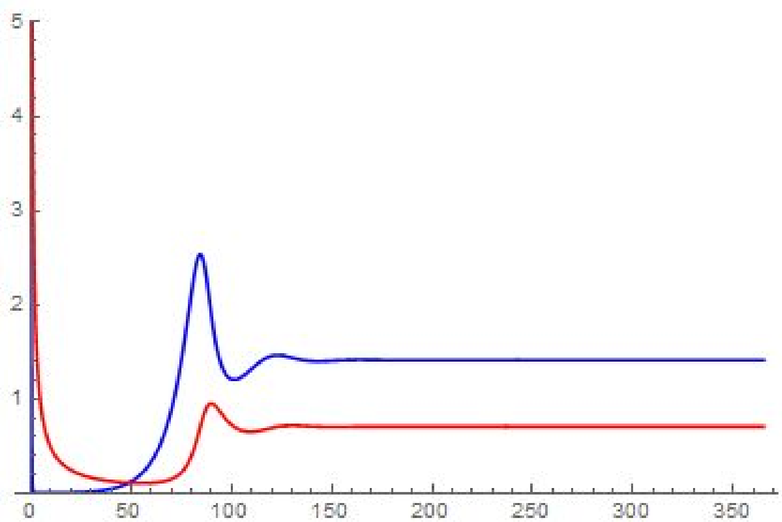

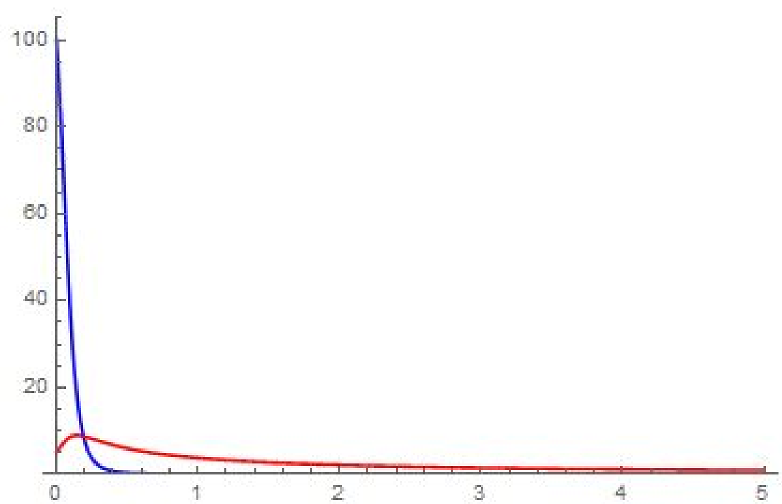

$$\frac{dx}{dt} = ax - bxy$$

$$\frac{dy}{dt} = -cy + dxy$$