

SCUDEM 2019

Problem C: Chemical Espionage

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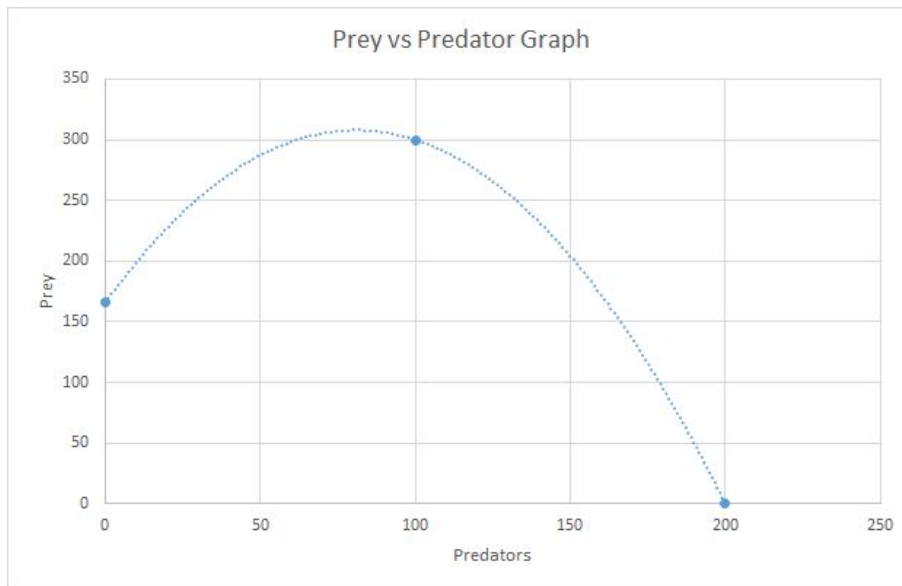
Problem: Establish the trade-offs and balance between two competing insects; the white cabbage butterfly, and the parasitic wasp species. Use a mathematical model for the relation of the insects. Lastly, determine what is the best balance for this system and what will occur in the long run.

Our approach: After some initial research about different methods of modeling populations, we first considered the Lotka Volterra Model. These set of differential equations related predator and prey relationships; however, it did not match the characteristics of wasps and butterflies as we had hoped. Our next approach involved mapping the relationship between the wasps and butterflies by using the four differentials below in Appendix A. With these four equations we were able to appropriately model the stability of the populations over a period of time taking into account different scenarios between the two species. One scenario modeled the relationship between the wasps and butterflies in a purely predator prey model, meaning that any kind of decrease in the wasp population would come from natural death. In the next scenario, we identified the relationship between the two as being competition, factoring in that as the butterfly population is decreased, the wasp population falls off as well.

Model:

(Appendix A)

Modeling of Predator vs Prey		
Predator Model	$\frac{dx}{dt} = -Ax + Bxy$	$\frac{dx}{dt} = -Ay + Bxy$
Prey Model	$\frac{dx}{dt} = Cy - Dxy$	$\frac{dx}{dt} = Cx - Dx^2 - Exy$
	Only affected by each other	Prey is affected by some factor (i.e carrying capacity)



Outcomes and Conclusions: After completing the study and formalatting models based on the data, it can be seen that in the long run both the species would fluctuate over time, but it will still maintain the pattern as the predator prey model we developed.

References:

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