Executive Summary: Chemical Espionage

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Male *Pieris brassicae* use anti-aphrodisiac hormones, after mating, to ward off other competing males which allows female to focus on producing their eggs in the most advantageous place, protected from the harassment of the other males[2]. Two parasitic species viz. *Trichogramma brassicae* and *Trichogramma evanescens* use anti-aphrodisiac to locate the eggs of mated female butterflies and lay their own eggs on the host butterflies' eggs. The larvae of those parasitoids feed on the eggs of the *P. brassicae*[7]. This model attempts to depict the population dynamics of large cabbage white butterflies(*P. brassicae*) under this competing interest dynamics.

Assumptions

Only the population of a single colony of *Pieris brassicae* is considered for analysis.The population dynamics of the colony is studied over the period of mating season. It is then extrapolated over extended period of time. Every wasp that mounts on mated female butterfly is assumed to successfully cling to the butterfly until it reaches the host's egg-laying sites. It is also assumed that wasps destroy all possible eggs at those sites. Intra-specific competition amongst the wasps is not taken into consideration in this model. In addition to that, the butterflies-both healthy and infected are assumed to be equally prone to attack by the parasitoid wasps.

The Model

The population dynamics between Pieris brassicae and the parasitic wasps, under such competing interests, can be modeled using the following system of differential equations:

$$\begin{aligned} \frac{dB}{dt} &= rB - f(B_f)\alpha_1 W - g(B_f)B_m\alpha_2 \\ \frac{dW}{dt} &= -dW + f(B_f)\gamma W \end{aligned} \tag{1}$$

where $f(B_f)$ and $g(B_f)$ are functional responses which are discussed in length in the section which follows. Also, B(t)= population of *Pieris brassicae* at time t, W(t)= population of parasitic wasps at time t, r= intrinsic growth rate of *Pieris brassicae*, B_f = Population of mated female *Pieris brassicae*, B_m = Population of male *Pieris brassicae*, α_1= number of eggs laid by one female *Pieris brassicae*, α_2= proportion of α_1 impacted by the harassment of male *Pieris brassicae*, d = death rate of the parasitic wasps, γ= conversion efficiency, number of parasitoids that emerge from parasitized *Pieris brassicae.*

Functional response, is defined as the relationship between the number of prey or hosts attacked by a predator or parasitoid as a function of prey density[4][6].The functional response of predator can be extended to parasitoids[3]. Type II functional response can be used for *Trichogramma brassicae* [1] in relation to the problem. Using the famous disc model for predators[4],

$$f(N) = \frac{aN}{1 + aT_h N} \tag{2}$$

where N is the number of prey, a the encounter rate while searching, and T. the handling time. When applied to parasitoid f(N) gives an encounter rate, i.e. the number of encounters with hosts per parasitoid per unit time[3]. For this problem, the functional response is the function of B_f and is influenced by the concentration of anti-aphrodisiacs produced.

$$f(B_f) = \frac{kB_f}{1 + khB_f} \quad (Wasp) \qquad g(B_f) = \frac{\beta B_f}{1 + \beta h_1 B_f} \quad (Male\, P.brassicae) \tag{3}$$

where,
k = is the encounter rate of parasitoid with the mated female, β= is the encounter rate of male with the mated female, h = handling time for the parasitoids, h_1= handling time for the males. k and β depend upon the concentration of anti-aphrodisiacs as anti-aphrodisiacs attract the parasitoids(k factor) and repel males(β factor). The relationship between k, β and c, factor incorporating the concentration of anti-aphrodisiacs is assumed

to be: k = μ_1c, $\beta = \frac{\mu_2}{c}$, where μ_1 and μ_2 are two constants.After replacing those value in (1), the following equations are obtained :

$$\begin{aligned}\frac{dB}{dt} &= rB - \frac{\mu_1 c B_f}{1+\mu_1 c h B_f}\alpha_1 W - \frac{\mu_2 B_f}{c+\mu_2 h_1 B_f} B_m \alpha_2 \\ \frac{dW}{dt} &= -dW + \frac{\mu_1 c B_f}{1+\mu_1 c h B_f}\gamma W\end{aligned} \tag{4}$$

Effect of Concentration of Anti-aphrodisiacs in the Model

Based on various researches conducted on the interaction between *Peris brassicae* and *Trichogramma brassicae*[1][2][5], the following observation can be made. Increase in concentration of anti-aphrodisiacs favors the growth of *Trichogramma brassicae* and causes the number of *Peris brassicae* to decrease. At the same time low release of the said hormone prevents the parasitoidal growth. It also increases the interference of other males on female during oviposition which hampers the nurturing of the eggs thus decreasing the birth rate and hence the population.During the extreme cases i.e. when $c \to 0$ the number of *Peris brassicae* increases rapidly and when $c \to \infty$ the number of parasitoids shows an accelerated growth. This model uses relevant data from various research to find the best-fit for our equations to model the population. The subject of interest here is to find the concentration factor that ensures balance in the population of the species. This occurs when $c \approx 0.00001$ units per female per day. The graph of the solution to the system of differential equations at this value of c was obtained using MATLAB.It is shown in figure 1 below.

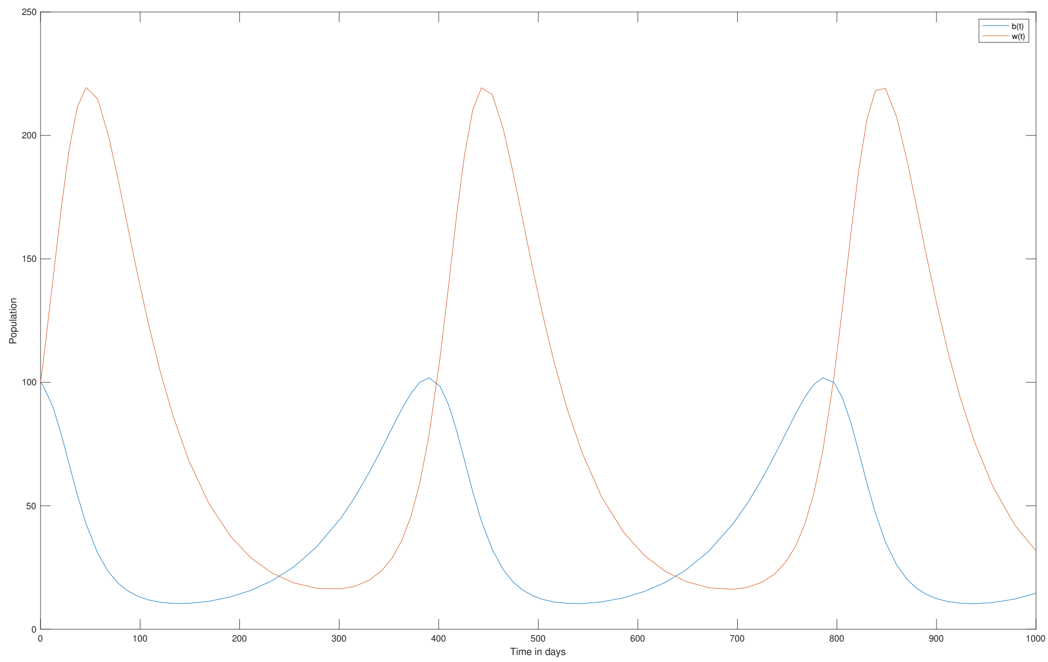

Figure 1: Population of the insects against time

Possible Improvements to the Model

Due to limited time, technical deficiency as well as for the sake of simplicity, some possible consideration are omitted during the formulation of this model. For instance, the mounting of of wasps on the mated female and the chance of other male to interfere them depends on a probability factor that is related to the concentration c. In this model, this probability factor is assumed to be linearly related to c for *Trichogramma brassicae* and inversely related to c for *Peris brassicae*.However utilizing a more sophisticated relation would make the model more convincing. In addition to that, this model forces the population of *Peris brassicae* to decrease below to negative values for significantly large value of c, which is not practically feasible. So improvement could be done to better model the population for the extreme values of c. Also species of wasps like *Trichograma evanescens* show different behaviour in response to the anti-aphrodisiacs as well as have different effect on the butterfly population which is not included in this model. Its inclusion would have represented the more accurate picture of the population dynamics.

References

[1] "A comparative study on the functional response of Wolbachia-infected and uninfected forms of the parasitoid wasp Trichogramma brassicae". English. In: *Journal of Insect Science* 10 (2010). ISSN: 1536-2442. DOI: 10.1673/031.010.14127.

[2] "Anti-aphrodisiac compounds of male butterflies increase the risk of egg parasitoid attack by inducing plant synomone production". English. In: *Journal of Chemical Ecology* 35.11 (2009). 09-3268, pp. 1373–1381. ISSN: 0098-0331. DOI: 10.1007/s10886-009-9714-5.

[3] R. Arditi. "A Unified Model of the Functional Response of Predators and Parasitoids". In: *Journal of Animal Ecology* 52.1 (1983), pp. 293–303. ISSN: 00218790, 13652656. URL: http://www.jstor.org/stable/4601.

[4] C. S. Holling. "Some Characteristics of Simple Types of Predation and Parasitism". In: *The Canadian Entomologist* 91.7 (1959), pp. 385–398. DOI: 10.4039/Ent91385-7.

[5] T. N. Manohar et al. "Demographic parameters of the indigenous egg parasitoids, Trichogramma spp., parasitizing the invasive tomato leafminer, Tuta absoluta (Meyrick) (Lepidoptera: Gelechiidae)". In: *Egyptian Journal of Biological Pest Control* 29.1 (Feb. 2019), p. 9. ISSN: 2536-9342. DOI: 10.1186/s41938-019-0112-1. URL: https://doi.org/10.1186/s41938-019-0112-1.

[6] M. E. Solomon. "The Natural Control of Animal Populations". In: *Journal of Animal Ecology* 18.1 (1949), pp. 1–35. ISSN: 00218790, 13652656. URL: http://www.jstor.org/stable/1578.

[7] Z.X. Wu, A.C. Cohen, and Donald Nordlund. "The feeding behavior of 6Trichogramma brassicae: New evidence for selective ingestion of solid food". In: *Entomologia Experimentalis et Applicata* 96 (Jan. 2000), pp. 1–8. DOI: 10.1023/A:1003923522482.