Pieris Brassicae vs Wasps

When a male Pieris Brassicae, a large cabbage white butterfly, mates with a female Pieris Brassicae, it often leaves anti-aphrodisiacs, a chemical signal, to protect the female that it mated with from other males. This chemical can be used by wasps to daetect a mated female butterfly, whose eggs can be exploited by wasps to host their eggs. If a male butterfly leaves anti-aphrodisiacs (AA) while mating, the population of butterfly will see a higher growth as female butterflies will be able to mate without many intrusions from multiple males. This tendency from male will also result in growth in population of wasps as wasps will have more chances of successfully detecting a female butterfly to lay eggs on butterfly's eggs. However, if a male does not leave anti-aphrodisiacs while mating, this will lead to decrease in both the butterfly and wasps population as more butterflies get subject to intrusions from multiple males while ovulating and more wasp will remain unsuccessful in detecting mated female butterflies to host their eggs on. This shows that the population of both butterfly and wasps depend on whether a male butterfly leaves anti-aphrodisiacs while mating.In addition, this shows an interrelation between population of butterfly and wasps, with butterflies being preys and wasps being predators. Our model explores all possible interactions among male and female butterflies as well as wasps to find the number of times female butterflies needs to receive anti-aphrodisiacs from male butterfly such that it will balance the population of butterfly and wasp and best benefits butterfly's population . The model is further extended to explore the possibilities of butterfly population over time.

Assumptions:

1. All wasp which mount on a female butterfly reach the host plant successfully.
2. Each egg (larvae) of the wasp eats (destroys) one egg of the butterfly.
3. Not all eggs laid by wasps on the host plant destroy butterfly eggs.Only the wasps eggs which hatch before the butterfly eggs hatch destroy the butterfly eggs.
4. Male and female butterflies each make half of the butterfly population.

Total Birth of Butterfly at Time 't', $B(t)$: A butterfly egg takes 7 days after ovulation to hatch. Therefore, the total number of successful birth of butterfly at time 't' is difference of the number of eggs laid by the butterfly during ovulation at time 't-7' and the number of butterfly eggs destroyed by wasps from the time the butterfly eggs ovulated to the time they hatched.

To find the total birth of butterfly, we consider contribution of mated female butterflies and wasps which come in interaction with AA and the ones which do not come in interaction with AA. Then, we get,

$$B(t) = B_{AA}(t-7) - D_{AA} + B_{\neg AA}(t-7) - D_{\neg AA}$$

B_{AA}(t) = Number of Eggs at time 't' Laid By Mated Female Butterflies Under the Influence of AA
D_{AA} (t)= Number of Eggs Destroyed by Wasps, Which Used AA For Host Detection, at time 't'
$B_{\neg AA}$ (t)= Number of Butterfly Eggs Laid at time 't' By Mated Female Butterflies Without Influence of AA
$D_{\neg AA}$ (t) = Number of Eggs Destroyed By Wasps, Which Did Not Use AA For Host Detection, at time 't'

Total Number of Eggs Laid By Mated Butterflies at Time 't', B_e (t):

$$B_e(t) = B_{AA}(t) + B_{\neg AA}(t)$$

The total number of butterfly eggs ($B_{AA}(t), B_{\neg AA}(t)$) laid by mated butterflies is a product of total number of ovulation by mated butterflies and the number of eggs laid per ovulation. The total number of ovulation of butterflies at time 't' is the product of the total population of mated butterflies and non-harassment coefficient.Non-harassment coefficient is defined to be number of harassment that a mated female experiences per ovulation. It provides a measure of proportion of the mated butterfly that is actually laying egg despite possible harassment from many males. This coefficient is greater for the butterfly who receive AA after mating than for the butterfly who do not receive AA after mating as female with AA are safeguarded from harassment by multiple males.

Therefore,

$$B_e(t) = NH1 * B_{matedAA}(t) * E_B + NH2 * B_{mated\neg AA}(t) * E_B$$

$$B_e(t) = NH1 * B_{matedAA}(t) * E_B + NH2 * (B_{totalmated}(t) - B_{matedAA}(t)) * E_B$$

Where, NH1=Non- Harassment Coefficient for Mated Female Butterflies Under the Influence of AA,$B_{matedAA}$= Population of Mated Female Butterflies Under the Influence of AA,NH2= Non- Harassment Coefficient for Mated Female Butterflies without Influence of AA ,$B_{mated}\neg AA$ = Population of Mated Female Butterflies without Influence of AA ,$B_{totalmated}(t)$ = Total Population of Mated Female Butterflies at 't',E_B = Total Number of Eggs Laid Per Ovulation

After exploiting logistic population change model of butterfly to find the number of mating female population at time t, our final equation for total number of eggs laid by mated butterflies at time 't' becomes:

$$B_e(t) = \frac{E_B * \gamma_{mating} * B_o K_B * (NH1 - D * NH2)}{(K_B - B_o)e^{\frac{-t*r_b}{K_b}} + B_o(D+1)}$$

Where, γ_{mating}= Total Mating Per Female ,B_o = Initial Population of Butterfly,K_B = Carrying Capacity of Butterfly ,r_b = Growth rate of Butterfly, D= Decision Factor = $\frac{B_{matedAA}(t)}{B_{mated\neg AA}(t)}$

Total Number of Butterfly Eggs Destroyed by Wasps at time 't', D_e(t):

$$D_e(t) = D_{AA}(t) + D_{\neg AA}(t)$$

A wasp egg takes 3 days after ovulation to hatch. By assumption 3,out of all the wasps eggs laid by wasps on the host plant, only the wasps eggs which hatch before butterflies' eggs hatch destroy butterflies' eggs. This implies the time period in which a wasp can ovulate an egg,which destroys butterfly eggs, starts at the time a butterfly starts ovulating (which is t-7 as explained in 3.1) and last until (t-4) as the wasp eggs which are born after t-4 do not have enough time to hatch before butterfly's birth and destroy it.For example: A wasp egg that ovulates on 5th (t-2) day after butterfly's ovulation can not destroy butterfly's eggs because the newborn wasp egg will need 3 more days to hatch while the butterfly's egg will hatch within 2 days.

Now, let $TE_n(t)$, $TE_e(t)$, $TB(t)$ be the population of naive T.Evanescens, experienced T.Evanescens and T.Brassicae wasps which ovulate at time t respectively. To find the total number of butterfly eggs destroyed by wasps at time 't',we multiply the total number of wasps ovulation from (t-7) to (t-4) with the number of eggs they can produce per ovulation (E_D).

After exploiting logistic population change model of wasp, we get,

$$D_e(t) = N_i(< M_b > + < M_o >) * \alpha_{TB} * W_{TB} * K_{TB} \int_{t-7}^{t-4} \frac{1}{(K_{TB} - W_{TB})e^{\frac{-t*\gamma_{TB}}{K_{TB}}} + W_{TB}} dt +$$

$$\frac{7}{20} N_i(< M_{TEe} > - < M_o >) * \alpha_{TEe} * W_{TEe} * K_{TEe} \int_{t-7}^{t-4} \frac{1}{(K_{TEe} - W_{TEe})e^{\frac{-t*\gamma_{TEe}}{K_{TEe}}} + W_{TEe}} dt +$$

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$$N_i(< M_o >) * \alpha_{TE} * W_{TE} * K_{TE} \int_{t-7}^{t-4} \frac{1}{(K_{TE} - W_{TE})e^{\frac{-t*\gamma_{TE}}{K_{TE}}} + W_{TE}} dt$$

γ = Total Mating Per Female of respective species, Ni = Total Number of Interaction of wasp with butterfly, $M_{species}$ = Mounting Coefficient of Wasp (The rate at which a wasp mounts on a mated female butterfly) , W = Initial Population of Species , K= Carrying capacity of the species,TB = T. Brassicae, TEe= Experienced T. Evanescence, TE= Total T. Evanescence

Final Total Number of Butterfly Eggs After Destructions by Wasps at time 't', $B(t)$

$$B(t) = B_e(t) - D_e(t)$$

Model Analysis:

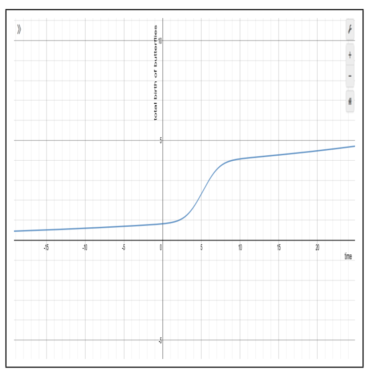

Figure 1: Total Birth of Butterflies, B(t), vs Time (t)

Our model shows that the total birth of butterfly is always greater than the total number of death of butterfly eggs because of wasps eggs. Since the model uses logistic population estimation model to predict population of butterfly and wasps over time, the birth of butterfly is increasing rapidly when there is abundant resources (carrying capacity of the butterfly) and is increasing slowly as carrying capacity decreases. We would get different results using our model if we used other population models to find change in population of butterfly and wasp, and hence, change in relation of their population, over time.

Conclusion: The model looks at birth of butterfly in two scenarios and calculates the total birth of butterfly at time 't'. In the first scenario, the total population of butterfly at time 't' depends on ovulation/destruction of butterfly eggs by mated female butterfly and wasps under the influence of anti-aphrodisiacs. In the second scenario, the total population of butterfly at time 't' depends on the ovulation and destruction of butterfly eggs by mated female butterflies/ wasps without the influence of anti-aphrodisiacs. The model individually looks into total birth and destruction of butterfly eggs over time in these two situations. Both the birth and possible destruction of butterfly eggs over time depends on the total population of butterflies and wasps, especially the population of female butterflies and female wasps at time 't'. So, our model is entirely based on logistic model of population estimation for estimating total population of butterflies and wasps over time in two situation described above. One of our model analysis presented in the report also shows a heavy effect of logistic model of population estimation. Use of a different model to make estimates regarding population of butterfly and wasps would give us a different result.

1 References

[1] “Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps, “ Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros. Behavioral Ecology. Volume 21, Issue 3, MayJune 2010, Pages 470–478, 11 February 2010. https://doi.org/10.1093/beheco/arq007 .