

Problem A: It's Hip to be Square

Alex Sawyer, Sam Snarr, Grant Wagner

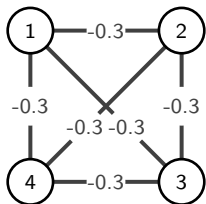
James Madison University Team 1; Coach: Roger Thelwell

November 14, 2019

Abstract

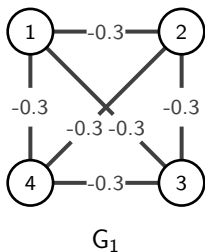
We develop a model for tracking the behavior of two groups, conformists (squares) and anti-conformists (hipsters), as they try to reach their respective style preferences through fad adoption. Fads are linked by a weighted graph, where the edge connecting fads i and j is a measure of how compatible i and j are. We develop a system of differential equations to generate the behavior of the fad popularity in the two groups over time.

A Fad Interaction Graph Example



G_1

A Fad Interaction Graph Example



$$A_1 = \begin{bmatrix} 1 & -0.3 & -0.3 & -0.3 \\ -0.3 & 1 & -0.3 & -0.3 \\ -0.3 & -0.3 & 1 & -0.3 \\ -0.3 & -0.3 & -0.3 & 1 \end{bmatrix}$$

The Model

$$\left\{ \begin{array}{l} \frac{ds_i}{dt} = \tilde{s}_i(1 - \tilde{s}_i) \left[\sum_{k=1}^N A_{ik}(\tilde{s}_k + \alpha \tilde{h}_k) \right], \tilde{s}_i(0) = \tilde{s}_{i0} \\ \frac{dh_i}{dt} = \tilde{h}_i(1 - \tilde{h}_i) \left[\sum_{k=1}^N A_{ik} \tilde{h}_k - \beta \tilde{s}_i + \frac{\gamma}{\tilde{s}_i} \right], \tilde{h}_i(0) = h_{i0} \\ \alpha, \beta > 1, \gamma > 0. \end{array} \right.$$

The Model

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s_i : number of squares following trend i

h_i : number of hipsters following trend i

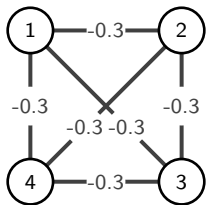
\tilde{s}_i : proportion of squares following trend i

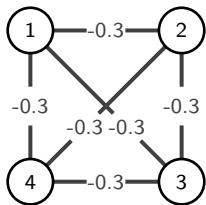
\tilde{h}_i : proportion of hipsters following trend i

α : parameter governing the weight squares place on hipsters' tastes (3.0)

β : parameter governing the aversion of hipsters to squares (3.0)

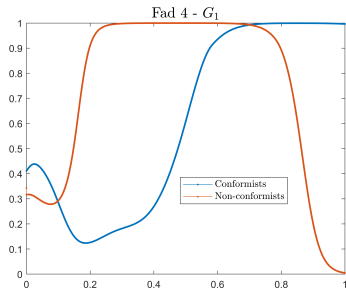
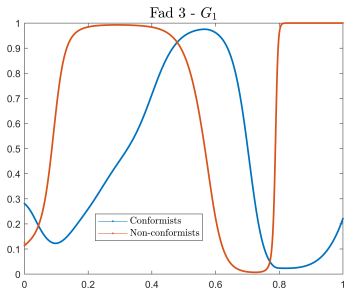
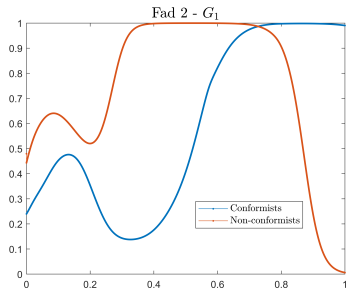
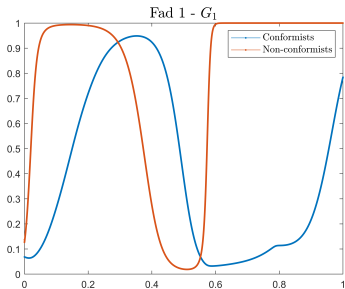
γ : parameter governing the desire of the hipsters to be contrarian (0.5)

G_1  G_1

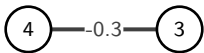
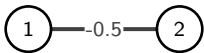
G_1  G_1

$$A_1 = \begin{bmatrix} 1 & -0.3 & -0.3 & -0.3 \\ -0.3 & 1 & -0.3 & -0.3 \\ -0.3 & -0.3 & 1 & -0.3 \\ -0.3 & -0.3 & -0.3 & 1 \end{bmatrix}$$

Plots for G_1 :

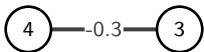
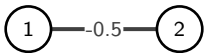


G_2 :



G_2

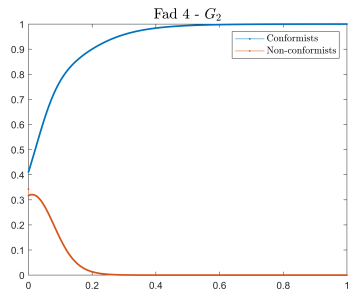
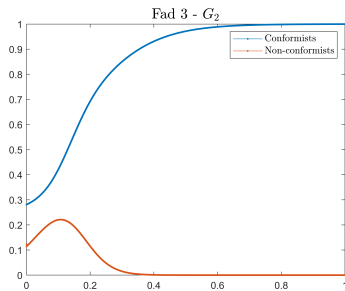
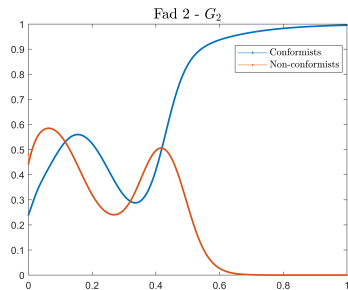
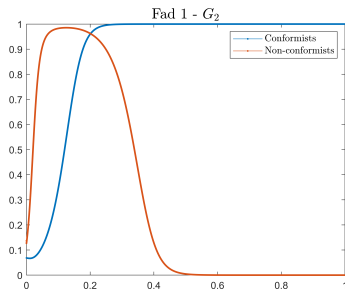
G_2 :



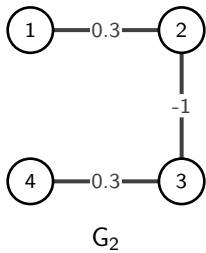
G_2

$$A_2 = \begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.3 \\ 0 & 0 & -0.3 & 1 \end{bmatrix}$$

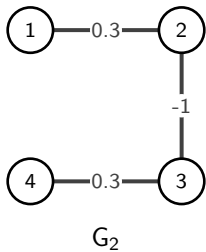
Plots for G_2 :



G_3 :

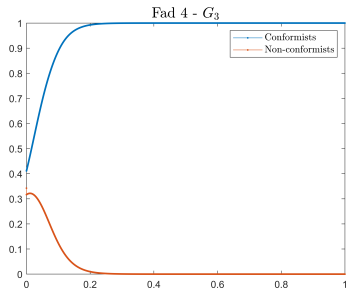
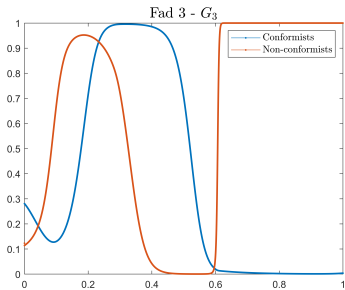
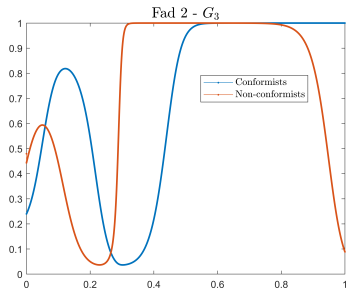
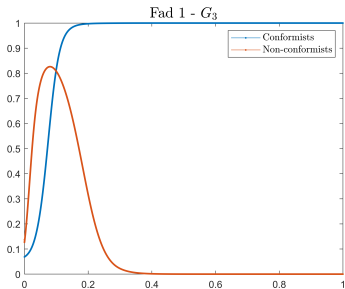


G_3 :



$$A_2 = \begin{bmatrix} 1 & .3 & 0 & 0 \\ .3 & 1 & -1 & 0 \\ 0 & -1 & 1 & .3 \\ 0 & 0 & .3 & 1 \end{bmatrix}$$

Plots for G_3 :



Additional Issue 1

If a company were making an item whose purchase was correlated to some fad within the population, their best strategy would be to market exclusively to a small group of hipsters. Since hipsters respond quite rapidly to fad adoption within their community, our hypothetical company would see a sharp uptick in sales as the hipsters rush to buy their product.

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After the hipsters adopt the given fad, their behavior would drive the squares to adopt it as well, leading to a slower, sustainable level of sales in the immediate future.

Works Referenced

David Smith and Lang Moore, "The SIR Model for Spread of Disease - The Differential Equation Model," Convergence (December 2004)

<https://github.com/sam-snarr/scudem-challenge>