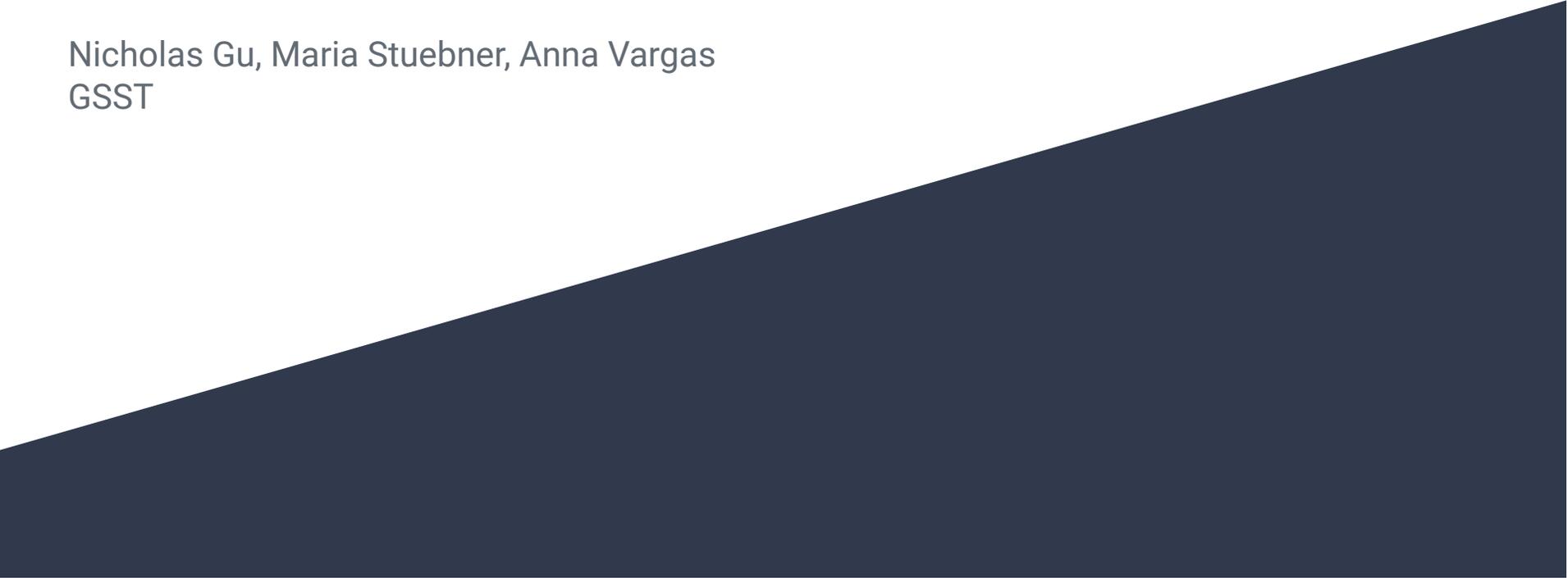


A Differential Equation Model to Simulate Trend Propagation Within a Population

Nicholas Gu, Maria Stuebner, Anna Vargas
GSST



Predict and assess the spreading of a trend within a population group: everything from similarity of members, number of people affected, methods of communication between members, and pace of spread.

Problem Statement

Assumptions

1. The trend within our population concerns physical products or articles of clothing that are deemed either desired or fashionable/chic.
2. The members of our group are either participants in the trend or bystanders who do not participate. This means that our model is binary. This same binary model was utilized by Touboul, and the reasoning for such an assumption is that most, if not all, individuals completely buy into a trend to complete a certain “look” with only slight deviations in color or model to accommodate for differences in income. These, however, do not affect the overall look that one adopts, as our next slides will demonstrate.
3. With the development of social media and the internet, we assume that communications between groups are instantaneous. However, social groups based on individual identity, such as race and gender may have differing fashion trends: males won't be wearing bikini tops.



An elaboration on the reasoning for the binary model: The pair on the right are *Fear of God* denim jeans that cost \$900, while the pair on the left are those from fashion retailer Pacsun and go second-hand for a mere \$25 on marketplaces such as Bump and retail for approximately \$50

Exchange of Information

With social media and the ubiquity of telecommunication, interactions between people are all at once instantaneous but also more frequent.

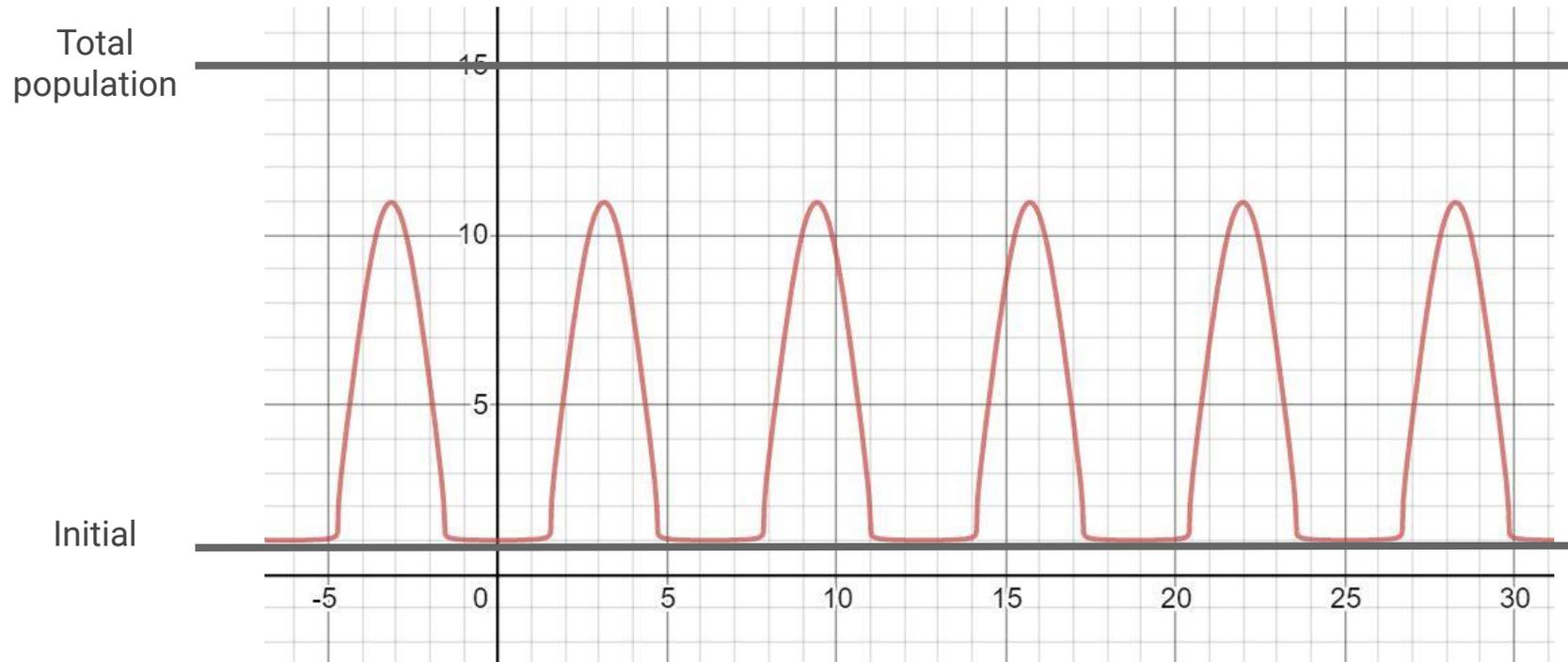
FOMO, otherwise known as the fear of missing out, is another phenomenon that would exacerbate the trends that we see. The peer pressures by people, especially those in the younger generation, are more pervasive and transcend other boundaries to include extenuating circumstances. Conformity is even more effective especially in cases of fashion which can be viewed by everyone whereas linguistic and material trends can be covered.

Differential Equation

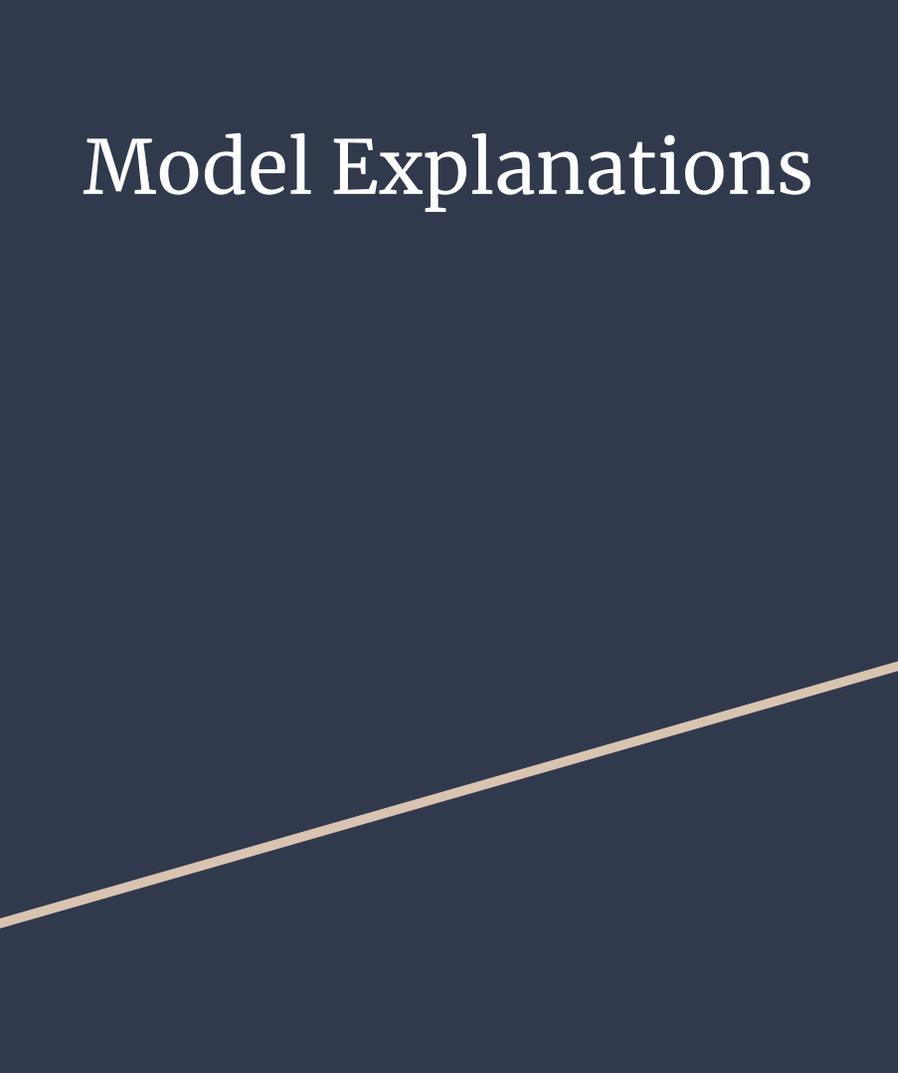
$$\frac{dp}{dt} = p \frac{\cos(t - \frac{\pi}{2})}{[\sin(t - \frac{\pi}{2})]^{\frac{2}{3}}}$$

$$p(t) = ke^{-3[\cos(t)]^{\frac{1}{3}} - 3} + 1 - \frac{k}{e^6}$$

Graph

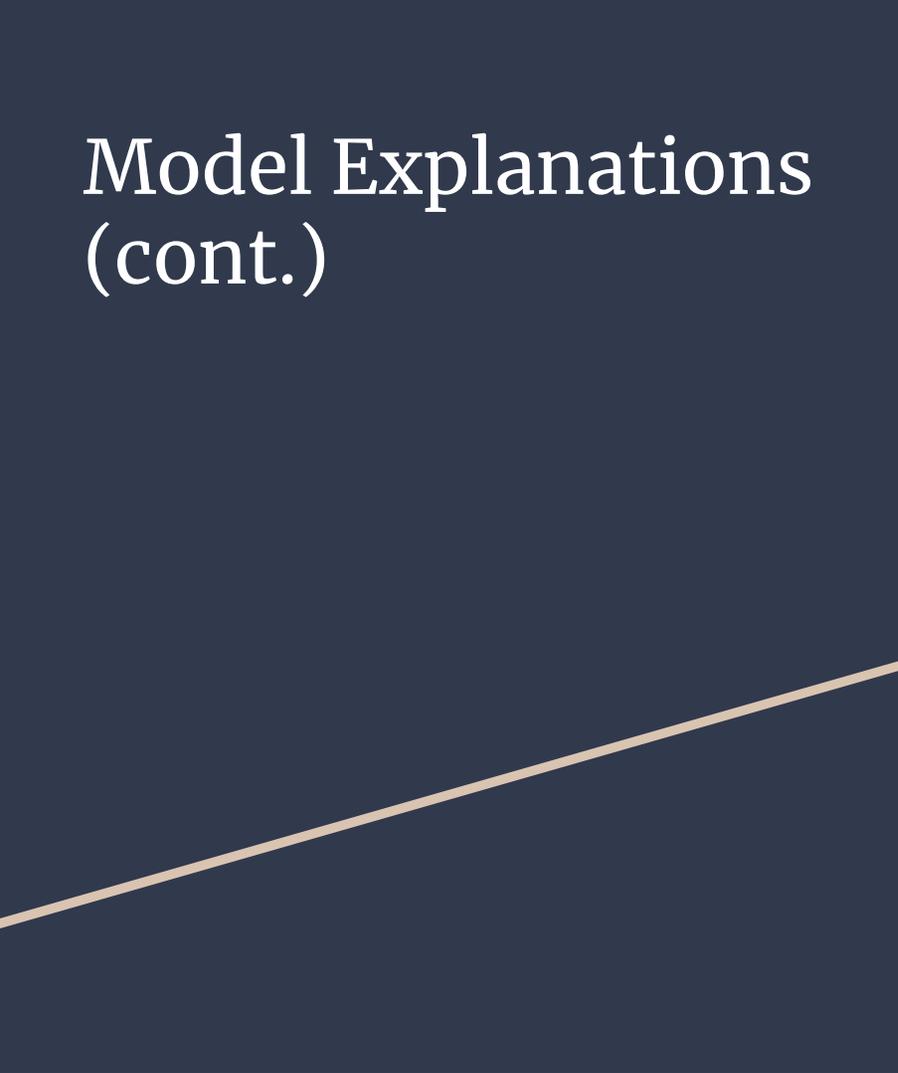


Model Explanations



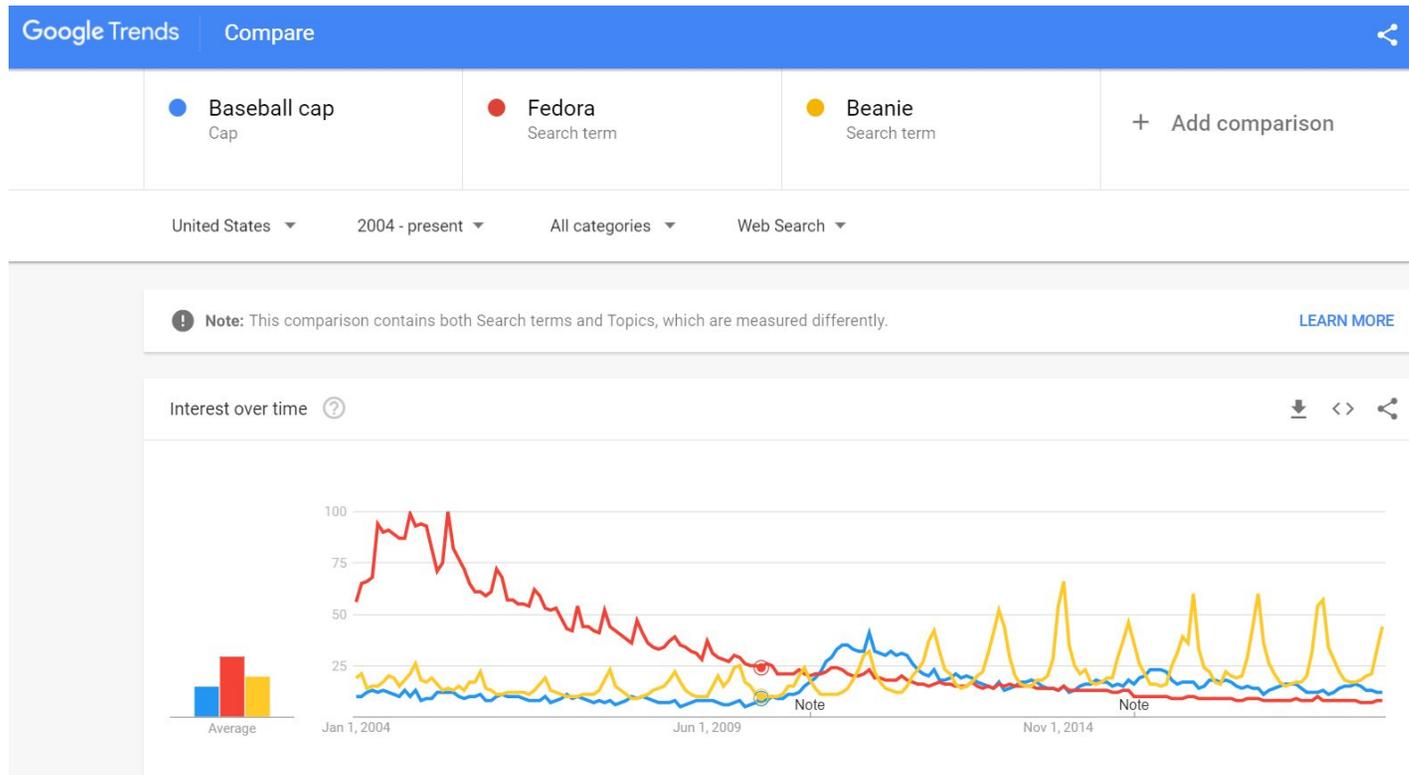
- The origin of the trend is a single individual, as with any trend, and the rate at which the population conforms increases over time. This is because as the number of people involved increases, the greater the influence due to greater peer pressure and chance of observation. When the maximum of the population have caught on to the trend, the trend reaches its peak and begins to fall after initial stagnation.
- The model shows a continuous rise of increasing conformity but does not reach the total population. This is an aspect of the equation that is advantageous to the model due to the reality that trends do not realistically affect the entire population. In any given time span, there continue to exist non-conformers.

Model Explanations (cont.)



- The reason for the fall of the trend over time can be attributed to the effects of time and the substitution effect.
- The substitution effect refers to the socio-economic effect in which a second trend or commodity emerges, stifling the first and creating an inverse relationship. This effect similarly applies in the model from which it can be inferred that the decrease in a trend over time (as opposed to immediate stagnation) is due to the external factors of such a substitutive occurrence or simply due to waning interest over time. (see example)

Google Analytics Example



Limitations

1. Our model would only be applicable to populations of middle to high income due to the differences in buying and conformity trends between social classes. However, this limitation can be mitigated due to the aforementioned assumption of low barriers of entry.
2. The rate may also differ based on social groups such as race and gender.
3. Since the model is oscillatory, utilizing a sinusoidal function, the peaks in trend interest are not realistic to an actual trend. Since the frequency is connected to the growth of one peak, it is not possible to account for both, and as a result, the limitations in frequency must be considered. Thus, we propose the first peak of the function, from $t = 0$ to $t = \pi/2$, to model the initial involvement in one trend, and we leave the frequency subject to change as each trend will come back in different intervals of time.

Limitations (cont.)

4. With smaller populations, the same type of mobility and propagation cannot occur. However, when there are larger groups, the model begins to more accurately portray the phenomenon.

Additional Issue: How does our model depend on initial conditions?

Using the initial values $p(\pi) = k$, where k is the total population:

$$p(\pi) = k = ce^{\left(-3[\sin(-\frac{\pi}{2})]^{\frac{1}{3}}\right)}$$

In order to account for an initial condition of $p(0) = 1$, representing the initial person to start a trend, we add a constant at the end of the equation:

$$1 - \frac{k}{e^6}$$

Since the model is oscillatory, the end behavior will always oscillate between $p = 1$ and $p = k + 1 - \frac{k}{e^6}$ from negative infinity to positive infinity since trends never die out; however, the frequency of the oscillations is limited due to the model chosen.

References

Marwan, K. M. (2002). *Globalization of Culture Through the Media*. (pp. 325–330). Philadelphia, PA: ScholarlyCommons.

Touboul, J. D (200X). *THE HIPSTER EFFECT: WHEN ANTI-CONFORMISTS ALL LOOK THE SAME*. (pp. 1-32). Waltham, MA. AIMS' Journals.

Julia Plevin. Who's a Hipster. The Huffington Post, 2008.

Paolo Dai Pra, Elena Sartori, and Marco Tolotti. Climb on the bandwagon: Consensus and periodicity in a lifetime utility model with strategic interactions. arXiv preprint arXiv:1804.07469, 2018.

Woolf, Jake. "The \$900 Jeans Rich Guys Can't Stop Wearing." *GQ*, GQ, 9 Aug. 2017, www.gq.com/story/fear-of-god-jeans-celebs.

PacSun Stack Skinny Jean. Pacsun, 2018, www.pacsun.com/pacsun/stack-skinny-jean-0132242100126.html?dwvar_0132242100126_color=510&cgid=mens-pants#prefn1=wash&prefv1=Light&start=21.