

Movement of An Object in Microgravity Environments

PROBLEM C

SCUDEM 2019

Ben Edwards, Yves Ngenzi, Keaton Brewster



Problem

The problem asks for the minimum dimensions the asteroid can have and the probe still be able to maneuver.

Constraints on types of maneuvers

Assumptions

The probe would break at 4 Gs.

No Coriolis Effect

There are no other large celestial bodies nearby

All space worms are friendly

Not being chased by the Empire

No air resistance



— Forces on a free falling probe motion in Asteroid Field

-when probe is close to a single asteroid, the only one force from the single asteroid will be relevant..

The only force acting on the probe is the gravity. .

$$F_g = GMm/(r+z)^2$$

Where r is the distance between the surface and the Center of Mass , and z is the radial distance from the surface. Note that it is not just the height, it has to account for non-spherical asteroids.

Limits on the Radius

Assumptions: suspension accelerated constantly over 1 meter

- is half as stable as the mars rover, so it can handle up to 40 m/s^2
- using kinematics, this gives a max impact velocity of 8.9 m/s .

Limits on the Radius

-Gave limits of $1676 < r < 2.6 * 10^7$ m on the radius

-while this was done assuming a sphere, the equations work out about the same for

And arbitrarily shaped asteroid, these forces just act from the CoM.



Probe rolling modeling/Check maple file

FB diagram (on the ground)

$F_b + mg - kx = ma$
 $-ma + F_b + mg - kx = 0$
 $ma - F_b - mg + kx = 0$
 $600 \frac{dy''}{dt} - b \frac{dy'}{dt} - \frac{G M_s M_p}{r^2} + 100y = 0$

$600 y'' - b y' + 100 y - \frac{G M_s M_p}{r^2} = 0$

$M_p = 600$
 $M_s = \int \text{Volume}$
 $M_s = \rho \left(\frac{4}{3} \pi r^3 \right)$

$b \rightarrow$ damping constant
 $b = 20$
 $k = 100$ (spring constant)
 $G = 6.655 \times 10^{-2}$
 $M_s = \rho$
 $M_p = 600 \text{ kg}$

to avoid crush of the probe

$$\frac{G M_p M}{(r+z)} = \frac{1}{2} m v^2 + \frac{K z^2}{2}$$

(probe in free fall motion) $\frac{2 G M_p M}{(r+z)} = m v^2 + K z^2$ (note resistance the probe is in free fall motion)

Assume $r \gg z$

Then

$$\frac{2 G M_p M}{r} = m v^2 + K z^2$$

$$\sqrt{\left(\frac{2 G M_p M}{m r} - \frac{K z^2}{m} \right)} = v$$

$$\sqrt{\frac{2 G M_p M}{r} - \frac{K z^2}{m}} = v$$

we assumed that $K = 100$

Assume $x_0 = 20 \text{ m}$

$$G = 6.67 \times 10^{-2}$$

Assume

$$M_p = \left(\frac{4}{3} \pi r^3 \right) \rho \quad \rho = 2800 \frac{\text{kg}}{\text{m}^3}$$

$v \leq \text{norm}$ if the probe is like more rough
 $\frac{v_1^2 - v_2^2}{2} = 2ad$
 $\frac{0 - v_1^2}{2} = -2ad$

Motion on the ground

swings under damped

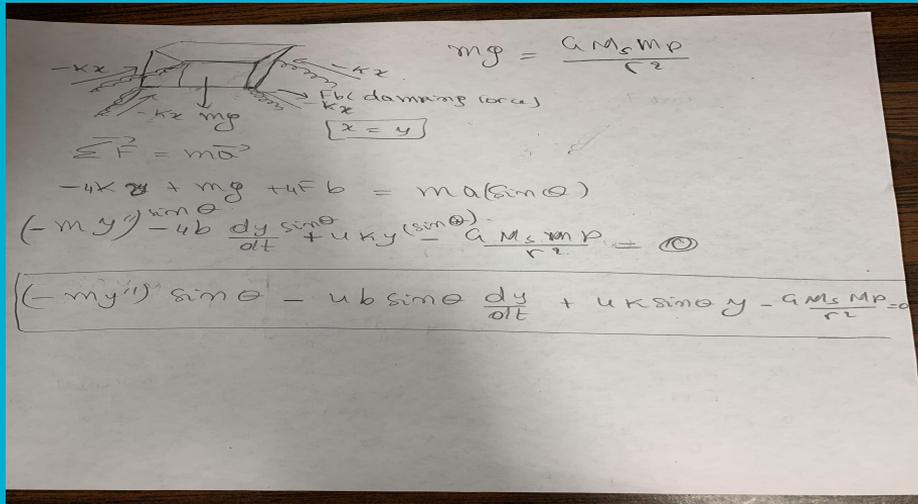
fixer of the spring

Conclusion

- we need to be careful on how we model the landing of our probe because it can crash
- Newton's law of motion will play an important role in our modeling
- Understanding the dimensions of the asteroid will be so important in choosing the way we have to land

Our model when our probe jumps once

It will do projectile motion to land to specified location



Constraints on valleys

-if the asteroid fits within the radial constraints, then the probe can jump arbitrarily high in any direction safely. Assuming that the suspension of the probe can absorb the entire impact and does so over 1 meter. These could be hydraulic or spring based.

-if the asteroid is larger than the radial constraints give, then it can jump in any direction as long as the final velocity is less than 8.9 m/s

-ideally, it should land on flat ground, but this is not a hard requirement, you could design the probe to avoid this

Method of Jumping

- Our probe has 4 springs, all oriented at 45 degrees from normal, and they are all 90 degrees from each other.
- The angles are constant but the springs can be loaded different amounts, allowing an arb. Velocity in any direction as long as the radial component is less than the escape velocity
- the energy from the springs is transferred instantaneously into the velocity of the probe, so the equation of motion is just dependent on the force of gravity

Equation for Jumping based on gravity

Used a numerical solution from maple, the a screenshot is shown here.

```
dsn := dsolve( [ z''(t) =  $\frac{-G \cdot M}{(r + z(t))^2}$ , z(0) = 0, z'(0) = 1 ], z(t), numeric, output = listprocedure )
dsn := [ t = proc(t) ... end proc, z(t) = proc(t) ... end proc,  $\frac{d}{dt} z(t) = \text{proc}(t) \dots \text{end proc}$  ]

G := 6.67 · 10-11; M :=  $\frac{2000 \cdot r^3 \cdot 4}{3} \cdot \text{Pi}$ ; r := 5000;

G := 6.670000000 10-11
M :=  $\frac{8000 r^3 \pi}{3}$ 
r := 5000

sol := eval(z(t), dsn); sol(1)

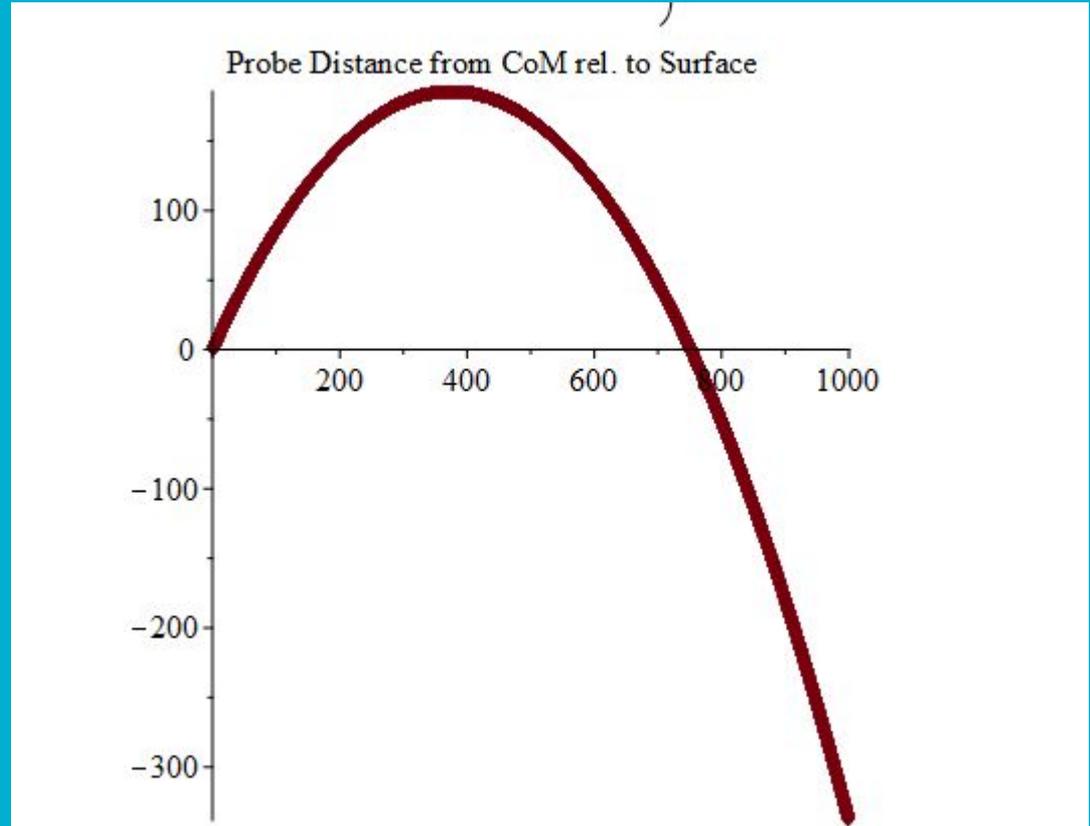
sol := proc(t) ... end proc
0.998603224570357

with(plots) : val :=  $\frac{\text{seq}(t, t = 0 .. 100000)}{100}$  ;
```

Equation for Jumping based on gravity

When approximating gravity to be constant near the surface, it is a parabola.

Note that while it is close to a parabola, the shape varies a bit, so it was worthwhile to include the actual acceleration



Equation for Jumping based on gravity

-Plot is radial height in meters

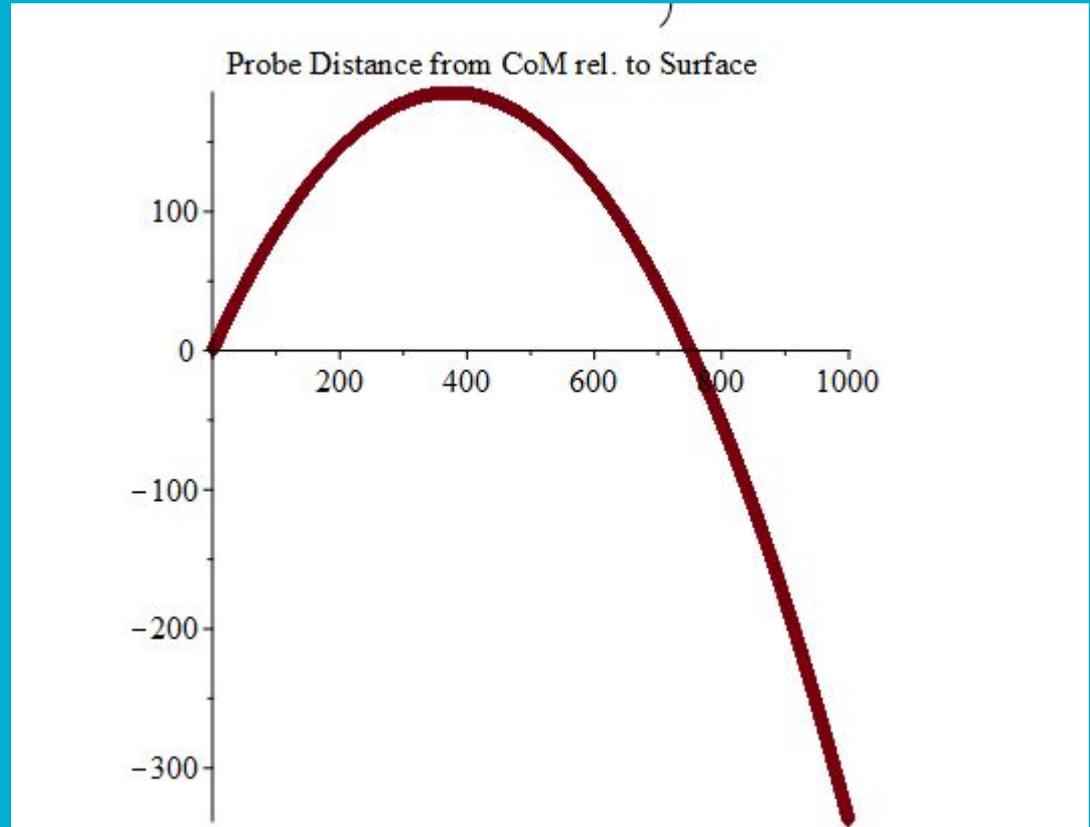
Vs time in seconds.

-Only force is in radial directions,

Other two orthogonal axes

Move at constant velocity for

Projectile motion



Equation for Jumping based on gravity

- This is a function of the radius Of the asteroid (assuming they All have similar densities), and The initial velocities.
- Too large of an initial velocity and The probe will escape gravity

