SCUDEM IV 2019
Problem B
Lincoln Memorial University

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Microgravity Movement

OBJECTIVES:

- Land a Probe on an Asteroid
- Determine Smallest Dimensions of Asteroid
- Develop a Method to Land at Predetermined Landing Spot
- Design a System to Move Probe on Asteroid

Method:

- Carrier Rocket is Launched into Space
- Rocket Orbits the Asteroid
- Probe is Deployed toward Asteroid
- Probe Uses Spring System to Travel

Lift-Off:

- Carrier Rocket is Launched
- Three-Stage-to-Orbit launch system is utilized
- $M\frac{dv}{dt} = v_{ex}\frac{dm}{dt}$ (momentum eq.)

Lift-Off:

When the function is integrated a constant exhaust velocity is achieved and we get that

$$v_f = v_i + v_{ex}\ln\left(\frac{M_i}{M_f}\right)$$

The formula given for a change in velocity is:

$$\Delta V = -c\ln\left[1 - \frac{(1-S)M_r}{(P+M_r)}\right] \quad (1)$$

with M_r equal to the mass of an engine, payload P, structural factor S, determined by the ratio of the rocket with fuel to the total mass of the rocket including the payload, and exhaust c, which will be called equation 1. $\Delta v = v_f - v_i$,$v_{ex} = -c$, $M_i = P + M_r$, and $M_f = P - SM_r$

This equation makes some basic assumptions in order for it to work properly.

If M_i is the mass of the i th stage, then it can be initially considered for the first stage that the rocket engine has mass M_1 and its payload has mass $M_2 + M_3 + A$. The second and third stages are handled similarly. Using equation 1 for ΔV and the mass of the third stage $(M_3 + A)$ it is found that the velocity after the third stage is jettisoned will be

$$V_3 = c\ln\left[\frac{SM_3 + A}{M_3 + A}\right]$$

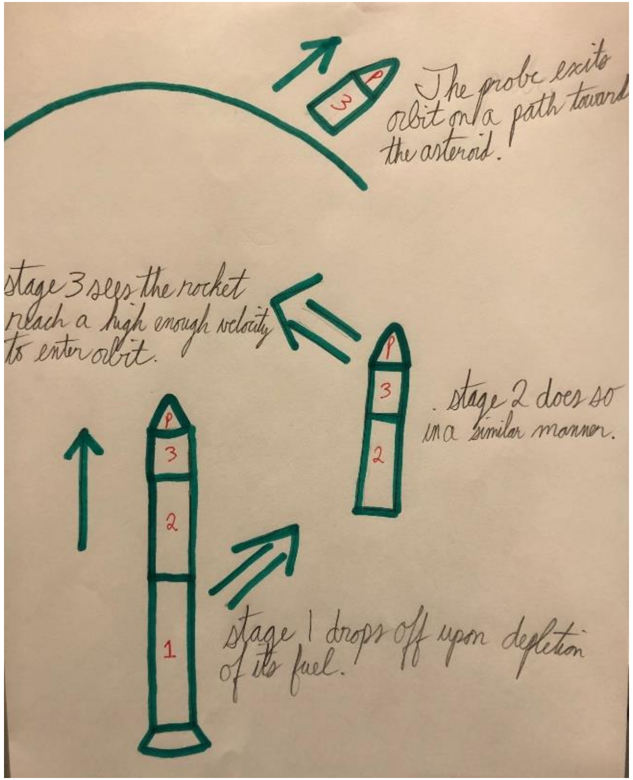
Lift-Off:

Using a similar process for the second and first stage it is possible to show that the velocity after all three stages have been jettisoned (equation 2) is:

$$v_f = c\left[\ln\left(\frac{M_1 + M_2 + M_3 + A}{SM_1 + M_2 + M_3 + A}\right) + \ln\left(\frac{M_2 + M_3 + A}{SM_2 + M_3 + A}\right) + \ln\left(\frac{M_3 + A}{SM_3 + A}\right)\right]$$

(1)

Rocket Launch Design



Orbital Motion:

- Carrier Rocket Orbits Asteroid
- Angular Velocity: $\dot{\vartheta} = \dfrac{\ell}{mr^2}$
- $\dfrac{d^2u}{d\vartheta^2} + u = -\dfrac{m}{\ell^2 u^2} F(\dfrac{1}{u}) \ \ , \ u = \dfrac{1}{r}$
- (3)

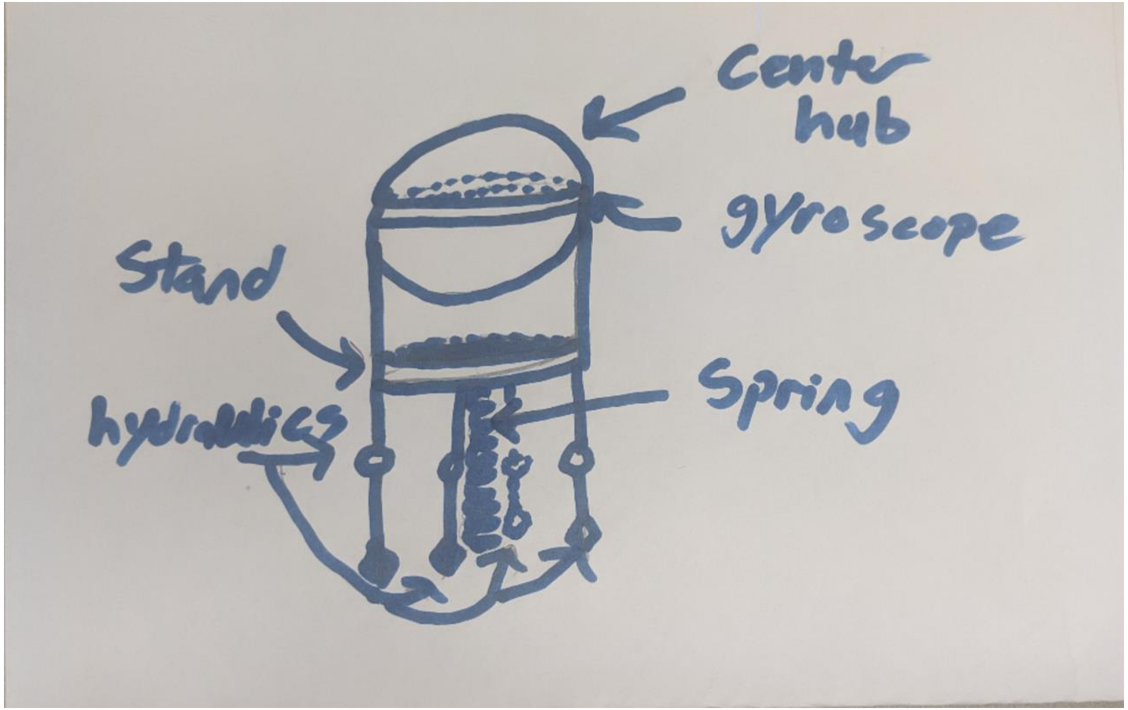
Probe Landing:

- Carrier Rocket Releases Probe
- Probe undergoes Free-Fall
- Newton’s Law of Gravitation

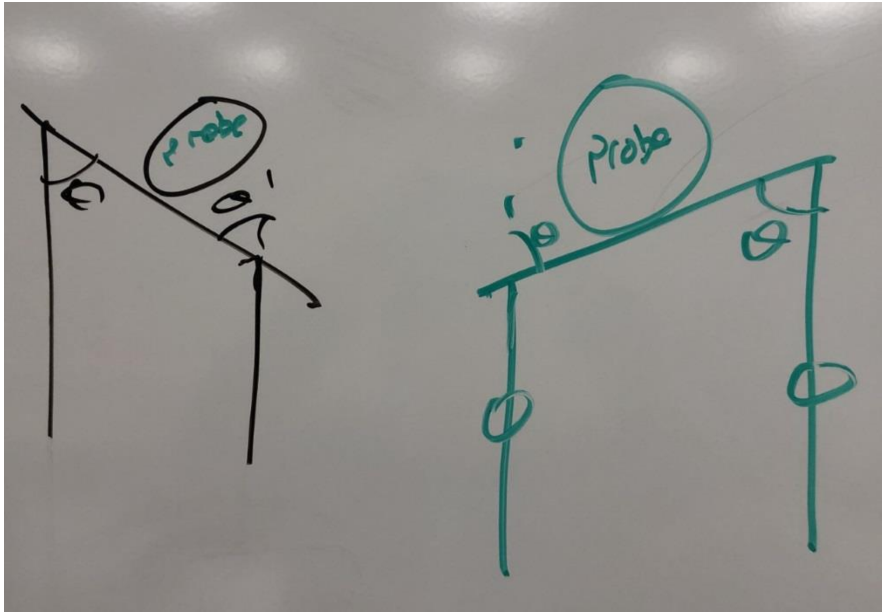
Probe Landing:

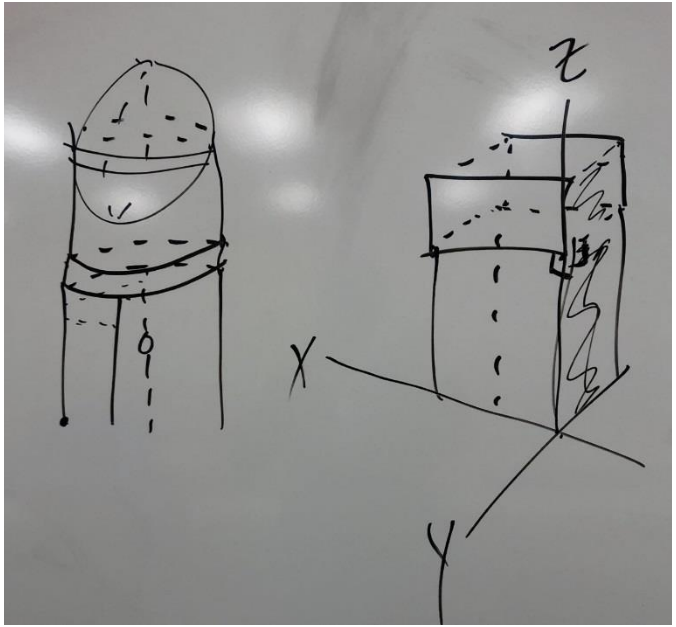
- Free-Fall:
- $\frac{d^2x}{dt^2} = g_{\mathcal{A}} - \frac{\mathcal{A}}{m}\frac{dx}{dt}$
- Landing Oscillations Model:
- $-mg = kx + m\ddot{x} - \mu m\ddot{x} + \sum b_n[cos\vartheta_n + sin\vartheta_n]m\frac{d^2x}{dt^2}$
- (2)

Probe Design



Probe Design





Landing Mechanics

- Probe Has a Rotatable Spring In the Event of Landing on Side.
- Gyroscope Used to Rotate Movable Spring into Position
- Low Gravity with Spring Shocks Protect Probe From Impact

Post-Land Travel:

- Spring is Positioned Using Gyroscope
- Probe is Propelled by Spring
- $-mg = kx + m\ddot{x} - \mu m\ddot{x} + \sum b_n[cos\vartheta_n + sin\vartheta_n]m\frac{d^2x}{dt^2}$
- Projectile Motion:
- $k(y_i - y_c)^2 + 2mgy_c = 2mgy_t + ky_t^2 + m(\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt})^2$

Additional Issue Question B #3

- Assume initial position of Probe = 0
- $\mathcal{P} = x(t) + y(t) + z(t)$
- $\frac{dP}{dt} = \frac{\delta x}{\delta t} i + \frac{\delta y}{\delta t} j + \frac{\delta z}{\delta t} k$
- $K(y_i - y_c)^2 + 2mgy_c = 2mgy_f + Ky_f^2 + m(\frac{\delta x}{\delta t} i + \frac{\delta y}{\delta t} j + \frac{\delta z}{\delta t} k)^2$

Limitations

- Probe Linear Path to Asteroid
- Low Gravity on Asteroid Implies Low Acceleration
- Probe Landing in Unescapable Trench
- Spring Losing Elasticity from Impact
- Radius of Probe Must be Much Smaller than Asteroid's.
- Rocket Has Low Range Due to Linear Flight Path

Questions?

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References

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- (2)Nagle, R. K., Saff, E. B., & Snider, A. D. (2019). *Fundamentals of differential equations*. Harlow, United Kingdom: Pearson Education Limited.
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- (4)Zill, D. G. (2013). *A First Course in Differential Equations with Modeling Applications, Tenth Edition / Dennis G. Zill*. Boston, MA: Brooks/Cole, Cengage Learning.

Thank you!