SCUDEM 2019 - Problem B

NYU Team 3
Members:Jacob Scott, Youngin Seo, Jason Zheng
Coach: Mutiara Sondjaja
New York University

Three parts of the question

Part 1: Landing the probe

Part 2: Moving the probe with springs, bouncing to a new location

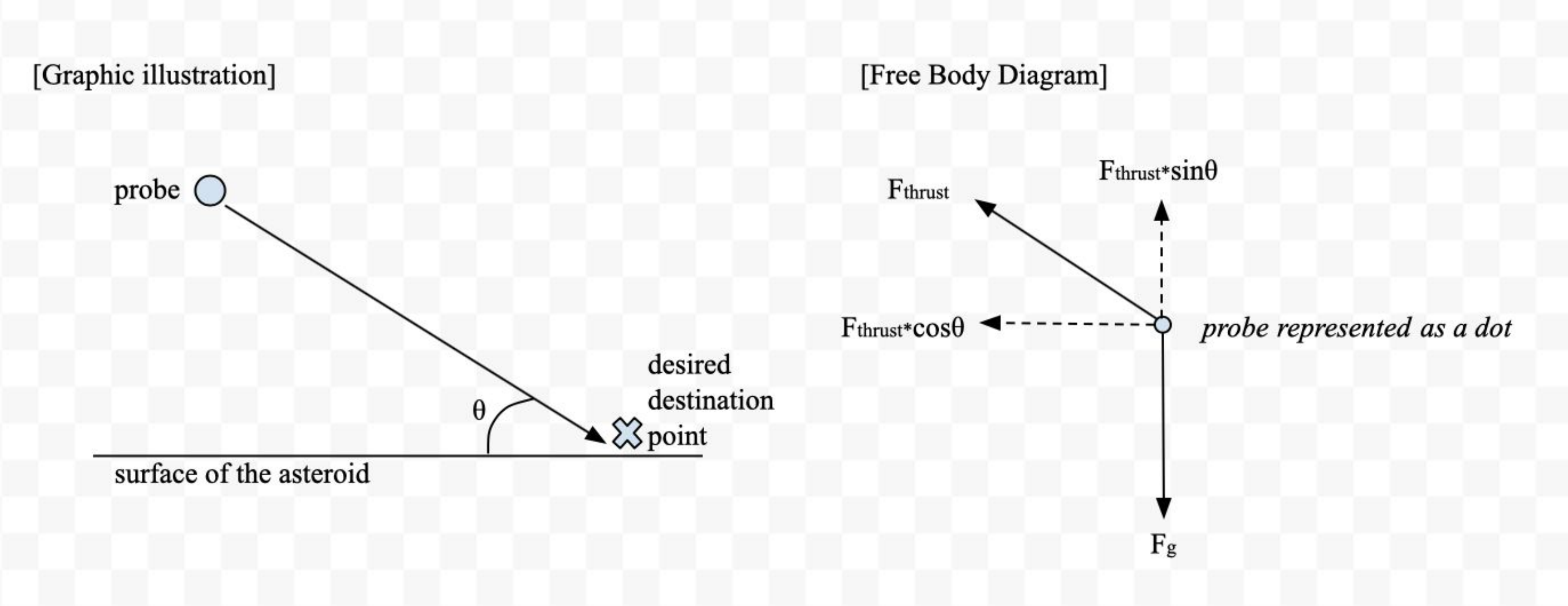
Part 3: Choosing the fit asteroid (Mass & Dimension)

Assumptions

- There is no Air Resistance because space is vacuum
- Gravity *g* is given and constant in the microgravity environment
- The asteroid is not moving relative to the probe
- Mass of the probe is given and constant
- Initial velocity v_0 is given and constant
- Probe landing sequence starts when $h_0 = y_0 = 10000$m
  - The height is given
- If v≤ (predetermined safe velocity), the probe will not be damaged
  - For purpose of calculation: Final velocity v is zero (both x and y axis)

Part 1 : Landing the Probe

All initial values come from when the distance between the probe and the asteroid is 10km, the probe initiates the landing sequence:



Part 1 : Landing the Probe - Y-axis

Starts with Newton's second law: $F = ma$

Consider only Y-axis $F = ma_y = F_{thrust} sin(\theta) - F_g$

$$\int \frac{dv_y}{dt} dt = \int a_{thrust} sin(\theta) - g dt$$

Divided both side by mass

$$\int_{v_{y0}}^{v} dv = \int_{0}^{t} a_{thrust} sin(\theta) - g dt$$

$$v_y - v_{y0} = a_{thrust} sin(\theta)\, t - g\, t$$

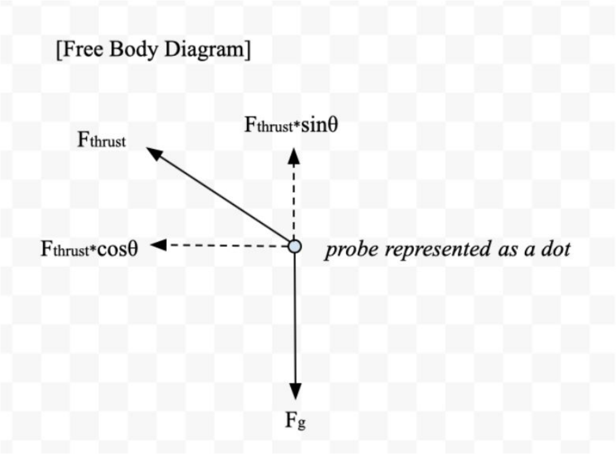
$$v_y = a_{thrust} sin(\theta)\, t - gt + v_{y0} = \frac{dy}{dt}$$

$$\int_{y_0}^{y} dy = \int_{0}^{t} a_{thrust} sin(\theta)\, t - gt + v_{y0} dt$$

$$\int_{y_0}^{0} dy = \int_{0}^{t} a_{thrust} sin(\theta)\, t - g\, t + v_{y0} dt$$

$$0 - y_0 = \frac{1}{2} a_{thrust} sin(\theta)\, t^2 - \frac{1}{2} g\, t^2 + v_{y0}\, t$$

$$y_0 = -\frac{1}{2} a_{thrust} sin(\theta)\, t^2 + \frac{1}{2} g\, t^2 - v_{y0}\, t$$



$$y - y_0 = \frac{v_y + v_{y0}}{2} t$$

$$t = \frac{-2y_0}{v_{y0}}$$

Finding t using kinematic equations

$$a_{thrust} sin(\theta) = \frac{y_0}{\frac{1}{2}\frac{4(y_0)^2}{(v_{y0})^2}} + g$$

Part 1 : Landing the Probe - X-axis

Similar Process to Y-axis:

Consider only X-axis $F = ma_x = F_{thrust}\cos(\theta)$.

$$a_x = a_{thrust}\cos(\theta) = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$\int_{v0}^{v} dv = \int_0^t a_{thrust}\cos(\theta)\, dt$$

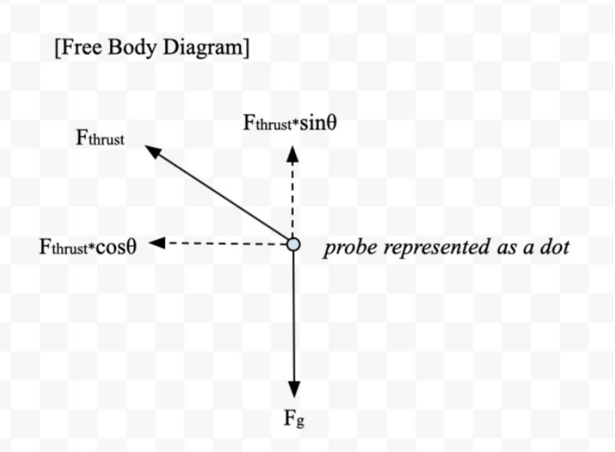
$$v_x - v_{x0} = a_{thrust}\cos(\theta)\, t - 0$$

$$v_x = a_{thrust}\cos(\theta)\, t + v_0 = \frac{dx}{dt}$$

$$\int_{x0}^{x} dx = \int_0^t a_{thrust}\cos(\theta)\, t + v_0\, dt$$

$$x - x_0 = \frac{1}{2} a_{thrust}\cos(\theta)\, t^2 + v_0\, t$$

$$x = \frac{1}{2} a_{thrust}\cos(\theta)\, t^2 + v_0\, t$$



$$x - x_0 = \frac{v_x + v_{x0}}{2} t$$

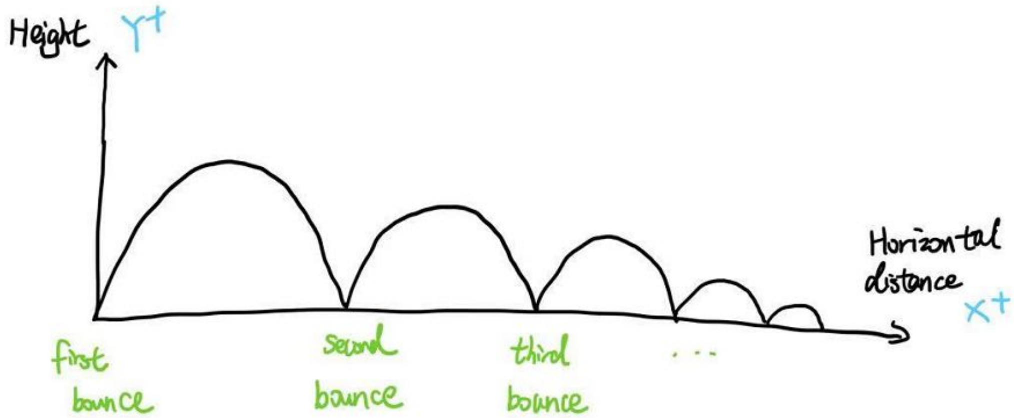
$$t = \frac{2x}{v_x}$$ Finding t using kinematic equations

$$a_{thrust}\cos(\theta) = \frac{(v_x)^2 - 2v_{x0}v_x}{2x}$$

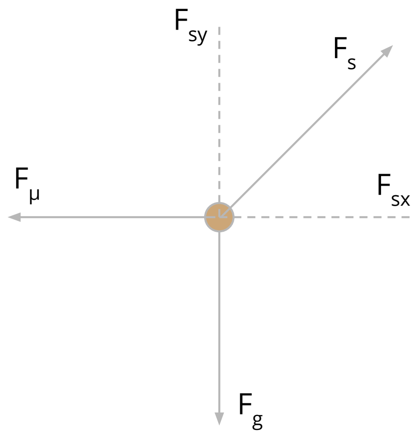
With acceleration of thrust and mass of probe known, we can solve the force of thrust in both X-axis and Y-axis.

Part 2 : Moving the probe with springs, bouncing to a new location

In this case, the probe has some spring force to make it bounce to a predetermined location:



[Free Body Diagram on First Spring force]



Part 2 : Moving the Probe-Conclusion

Sum of forces in bounces:

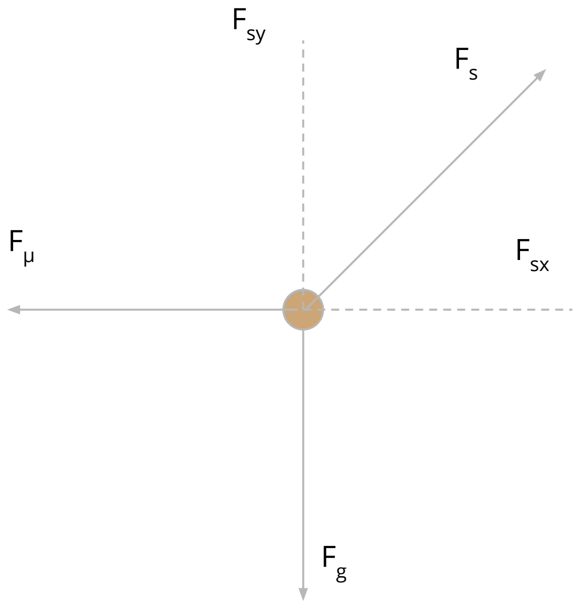
$$F_{total} = (F_1 + F_2 + F_3 + ... + F_n)$$

We also assume:

$$F_{s_n} = \mu F_{s_{n-1}}$$

$$F_{s_n} = (F_{s_1})(\mu^n)$$

[Free Body Diagram on First Spring force]



Part 2 : Moving the Probe- Conclusion

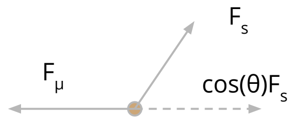
In this part, we only consider X-axis because our ultimate goal is to

let the probe move to a new position in x-axis.

$$V = \frac{dx}{dt}$$

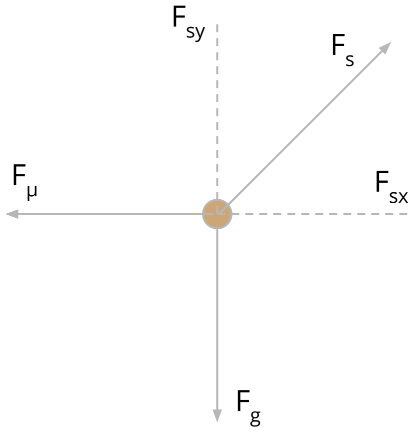
$$\frac{dv}{dt} = \frac{F_s cos(\theta) - F_\mu}{m}$$

$$x = \frac{1}{2} a_s cos(\theta)\, t^2 - \frac{1}{2}\mu g t^2 + v_0 t + x_0$$



Given x as a predetermined destination, with this formula, solving for F_s gives the spring force needed for the first bounce.

[Free Body Diagram on First Spring force]

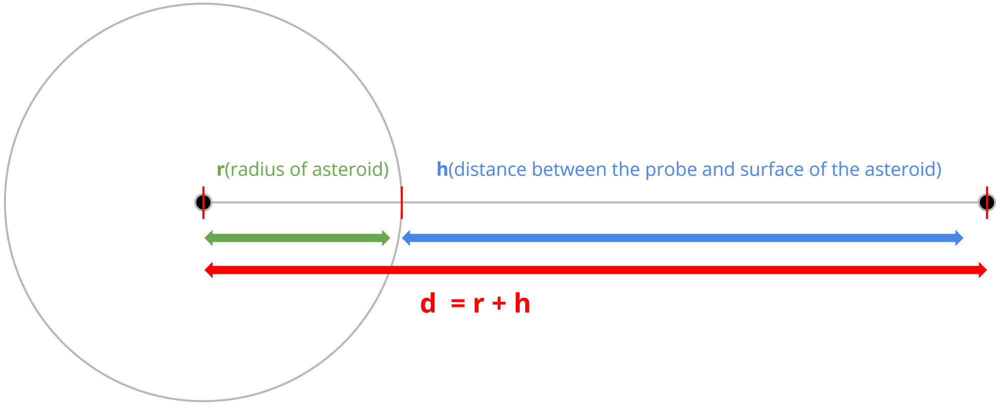


$$v = v_0 + at$$

$$t = \frac{v - v_0}{a}$$ Finding t using kinematic equations

$$X = \frac{(v - v_0)\left[(v - v_0) - \frac{F_\mu (v - v_0)}{F_s cos(\theta)} + 2v_0\right]}{2a} + X_0$$

Part 3 : Choosing the asteroid



Assume d = 10km = 10000m

$$F_g = G\frac{(m_1)(m_2)}{d^2}$$

$$\frac{d^2 F_g}{G m_1} = m_2$$

$$\frac{(r+h)^2 F_g}{G m_1} = m_2$$

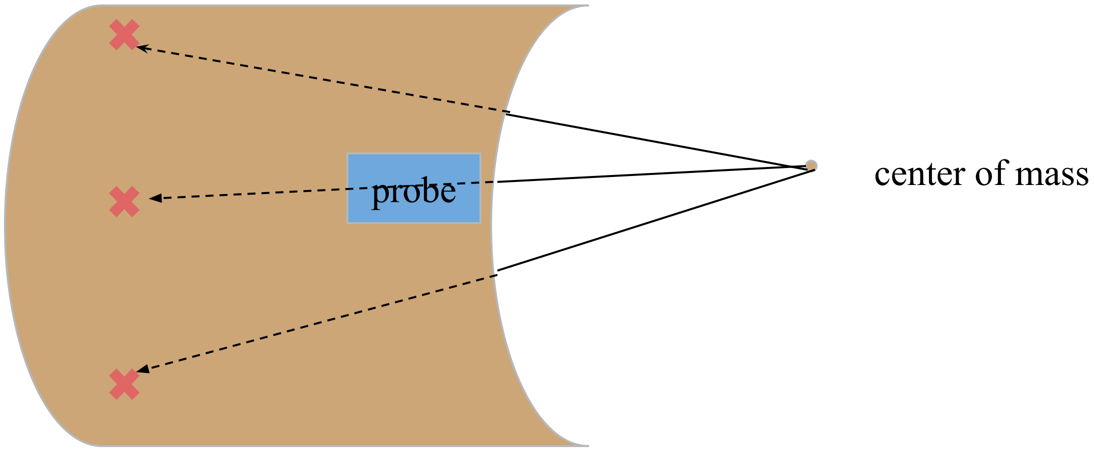
Note: F_g is solved in part 1

Limitations

- Gravity g is not constant in reality, it changes slightly at different positions of asteroid because of unequal distribution of mass and height above the surface.
- Mass of the probe, m1, is not always constant because of burning fuels.
- Neglected motion of asteroid relative to probe.

Part 4: Additional Issue

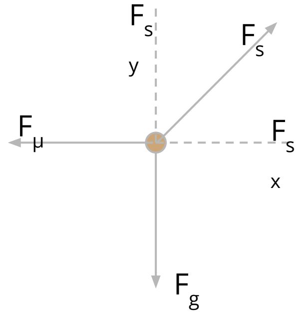
- Question: determine a way to make a probe travel around the circumference of an asteroid after it has landed, determine the best way to move along a given path and stay as close as possible to the path



Part 4: Additional Issue (cont’d)

Now that we know how the probe can navigate the ‘straight path’ to travel the circumference of the probe, there are three possible scenarios.

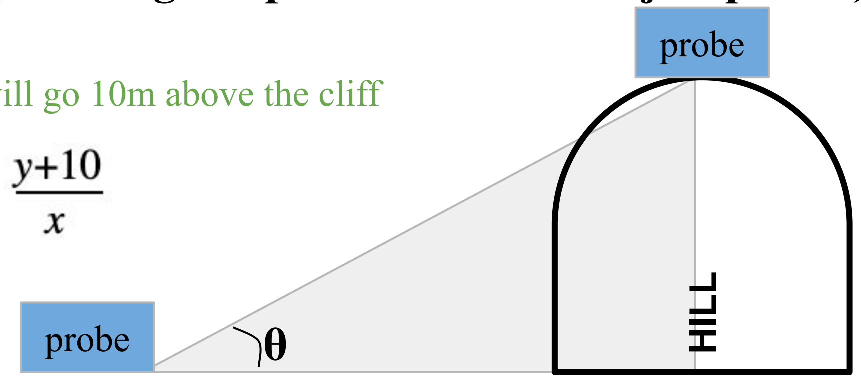
[Free Body Diagram on First Spring force]



1. Plain
2. Small hill (meaning the probe can easily jump over the hill)
   - Only need $F_s sin(\theta) > F_g$
3. **High hill (meaning the probe is unable to jump over)**

Assume the probe will go 10m above the cliff

$$tan(\theta) = \frac{y+10}{x}$$



x=distance to hill
y=height of the hill