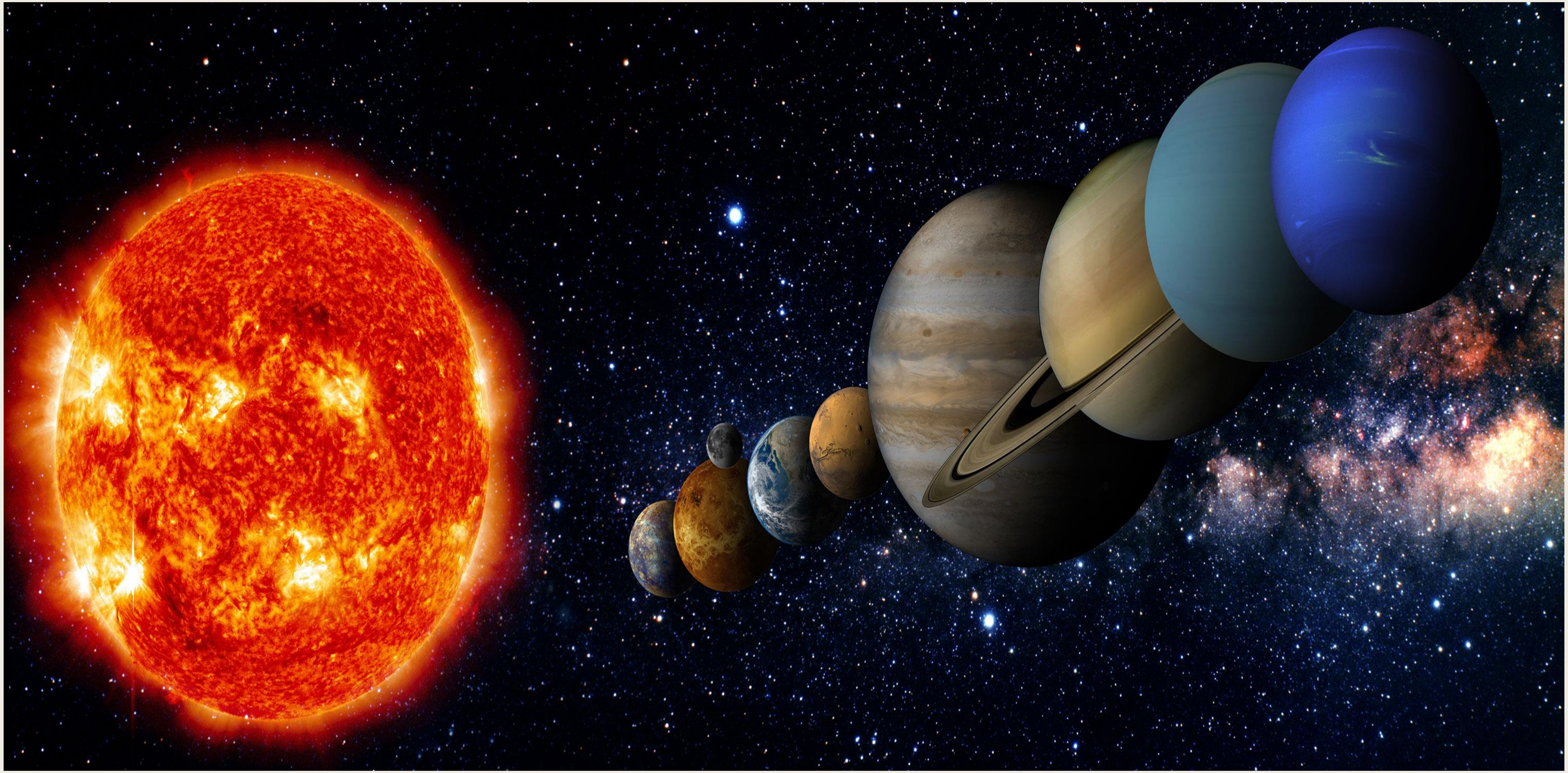
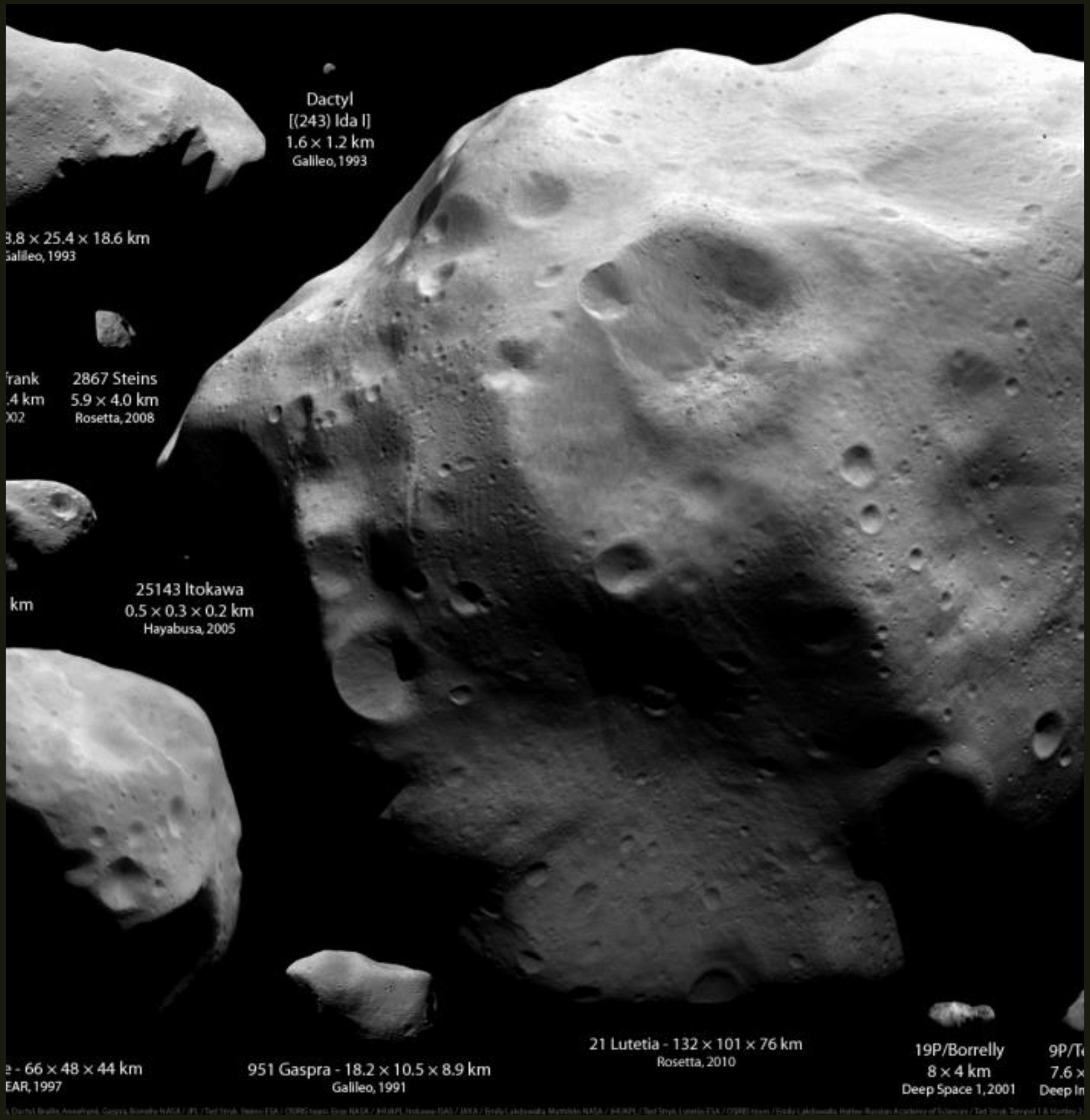
PROBLEM B: MOVEMENT OF AN OBJECT IN MICROGRAVITY ENVIRONMENTS

Dipu Biswas
Autumn Flores
Gladden Chukwu
Supervisor: Dr. Reza Ahangar

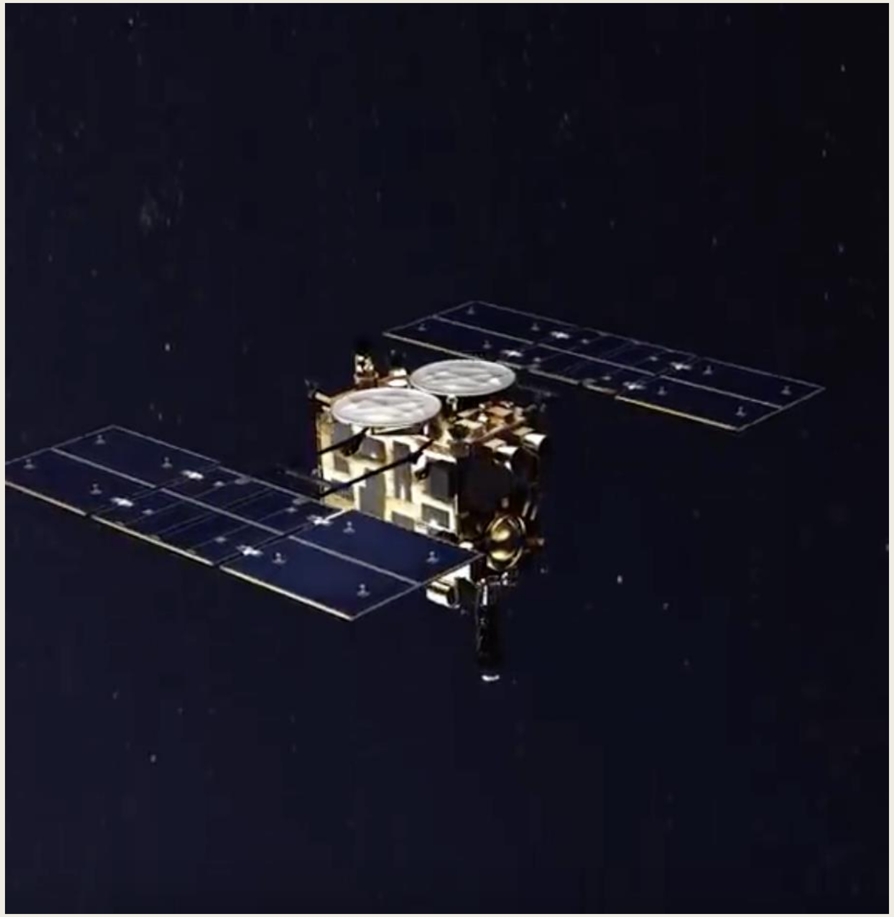
Introduction





Asteroid

- An asteroid is a rocky body of varying shape that orbits the sun
- They are made of clay or metals such as platinum, iron, and nickel
- They are remnants from the formation of the solar system



PROBE

A sophisticated spacecraft that is used to collect scientific information

Underlying Questions

- Determine the coordinates of landing for the probe
- Determine a way to move the probe on the asteroid using a spring
  - Using minimal amount of energy
- Determine the minimal amount of bouncing for the probe

Questions and Possible Solutions

Questions

- Coordinates of landing
- Minimal kinetic energy
- Minimal Amount of Bounce

Possible Solutions

- Landing Model
- Minimal Energy equation
- Bounce Model

Assumptions and parameters

- Assume hyperbolic landing, probe does not crash on surface of asteroid
- Assume that there is no air resistance
- r is the distance from probe to the landing point on the asteroid, but changes on time
- F_G= gravitational force of the asteroid
- G=gravitational acceleration of the asteroid
- a= acceleration of the probe
- m= mass of the probe
- M=mass of the asteroid
- v=velocity of the probe

Landing Model

- Newton's Gravitational Law
  - $F_1 = \frac{G*mM}{r^2}$
- Newton's 2^{nd} Law
  - $F_2 = m*a$
- $F_2 = ma = \frac{mv^2}{r}$

- Force acting on probe that causes a centripetal acceleration
  - $a = \frac{v^2}{r}$
- Force acting on probe
  - $F = \frac{G*mM}{r^2}$

Landing Model cont.

- Now F_1 = F_2
- $\frac{G*mM}{r^2} = \frac{mv^2}{r}$
- So, v= $\sqrt{\frac{Gm}{r}}$ = velocity of the probe
- But, as the probe will be closer to the asteroid, r changes

Landing Model cont.

- So, vector form of the Newton's Gravitational Law, F_1 =G$*\frac{mM}{r^2}* \vec{r}$
  - *where $\vec{r}$ is the position unit vector*
- F_1 is the vector form of force
- r=$\sqrt{x^2 + y^2}$
  - *where x= x coordinate*
  - *y= y coordinate of the position vector*

Landing Model, cont.

- Now, Components of the F_1 are
  - $(F_{1x}, \ F_{1y})$ = G$* \frac{mM}{x^2+y^2} * (\frac{-x}{\sqrt{x^2+y^2}})$, G$* \frac{mM}{x^2+y^2} * (\frac{-y}{\sqrt{x^2+y^2}})$
  - F_{1x} =m$* a_x$ = $-$G $* \frac{mM\ x}{(x^2+y^2)^{3/2}}$
  - F_{1y} =m$* a_y$ = $-$G $* \frac{mM\ y}{(x^2+y^2)^{3/2}}$

Landing Model, cont.

- Forward Euler's method of numerical integration of ordinary differential equations
- y(x_0)= y_0
  - *let* x_1 = x_0 +*h*
  - x_2 = x_0 +*2h*

where h= positive increment (step size)

- Now, if y_i represents the approximation of x_i, then
  - y_{i+1}= y_i +*h f(*x_i , y_i *)*
  - *h*= $x_{i+1} - x_i$

Landing Model, cont.

- To find the acceleration in x and y direction in the hyperbolic trajectory path of the probe, we can use the Forward Euler's Method
- $a_x = \frac{d^2x}{dt^2} = \frac{-u\,m\,x}{(x^2+y^2)^{3/2}}$ where μ=gravitational parameter of the probe
- $a_y = \frac{d^2y}{dt^2} = \frac{u\,m\,y}{(x^2+y^2)^{3/2}}$
- So, the continuously changed position of the probe with respect to the landing position on the asteroid will be
  - $r_{i+1} = \sqrt{(x_{i+1})\verb|^|2 + (y_{i+1})\verb|^|2}$

MATLAB Code to find the differential equations

```
%% Landing of the probe
%%h=dt
dt=50;
i=1;
while r<RSOIE && r>RE
t(i+1)=t(i)+dt;
vx(i+1)=vx(i)+dt*dvxdt(x(i),y(i));
x(i+1)=x(i)+dt*dxdt(vx(i));
y(i+1)=y(i)+dt*dydt(vy(i));
R(i+1)=sqrt(x(i+1).^2+y(i+1));
v(i+1)=sqrt(vx(i+1).^2+vy(i+1).^2);
psi(i+1)=atan2(y(i+1), x(i+1));
r=R(i+1);
i=i+1;
end
```

Bounce Model (minimum distance)

- Let h is the initial height (from bottom to top of the bounce) after bouncing for the first time after landing.
- 2h covers the first bounce. In the second bounce, a distance of 2hf is covered, and in the 3rd bounce 2hf^2 is covered, and so on.
- Total distance, x= 2h+2hf + 2h f^2 +…
- Geometric series
  - $X=\sum_{n=1}^{\infty} a * r^{n+1}$
- $X=\frac{2h}{1-f}$
  - *where, a=2h and r=f*
  - *X is the total distance by bouncing after landing*

Bounce Model (minimum distance), cont.

- Let the angle of bounce is θ
  - *The horizontal distance covered by the probe by bouncing*
  - $x = (v_0 * cos$ θ)t
  - The vertical distance covered by the probe
  - $y = y_0 + (v_0 * sin$θ)t+$\frac{1}{2}*at^2$
- Where
  - v_0 = initial bouncing
  - θ = bouncing time
  - t = time

Minimal Energy Equation (required to move the probe)

- Let x=distance of the spring to get it’s equilibrium position
- Energy needed to move the probe after landing
- Hooke’s law
  - $F=k*x$, *where k=spring constant*
- Potential energy of spring
  - $PE_{Spring}=\frac{1}{2}*k*x^2$
- Force applied in y-direction
  - $\sum F_y = -k*x-m*a=0$
- So,
  - $-k*x=m*\frac{d^2x}{dt^2}$ $\qquad K=-\frac{m}{x}*\frac{d^2x}{dt^2}$ = value of spring constant
- Total minimum energy to move the probe= kinetic energy + spring potential energy
  - $=\frac{1}{2}mv^2+\frac{1}{2}*k*x^2$

Conclusion

- The surface of the asteroid is assumed to be quite rugged. So, the probe will land in an angle with the asteroid.
- Coordinates of force can find the acceleration and velocity of the probe.
- The probe landed on the asteroid in a hyperbolic trajectory because velocity has a greater effect on probe than the distance covered on the trajectory path.
- After landing the probe on the asteroid, the probe will bounce with help of the spring with a minimum energy until it comes to rest.

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