Chemical Espionage -Problem C

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Background

- It is often hard for butterflies to find mating partners.
- One common way for female butterflies to find mates is by releasing chemical signals
- When a male butterfly senses this signal, it releases its own signal in response (called anti-aphrodisiac)
- This is done in order to dissuade other male butterflies from mating with these female butterflies
- Unfortunately, this chemical has the unintended side-effect of attracting parasitic wasps, hence the term chemical espionage
- These parasitic wasps that can sense this signal will often ride on a female butterfly to the location of the butterfly eggs, and lay its own eggs there

Background

Male butterfly infests female butterfly with anti-aphrodisiacs, thus helping impose female monogamy

Other males are dissuaded from pursuing the infested female butterfly, as a result of which the butterfly can focus on finding the most suitable spot for egg-laying

Wasps that can sense this chemical then follow these butterflies and lay their own eggs in these butterflies' nests

Host species


Butterfly eggs


Parasite species

*The butterfly species mentioned here is Pieris brassicae.
*The wasp species mentioned here is *Trichogramma brassicae*

Redefining the problem statement

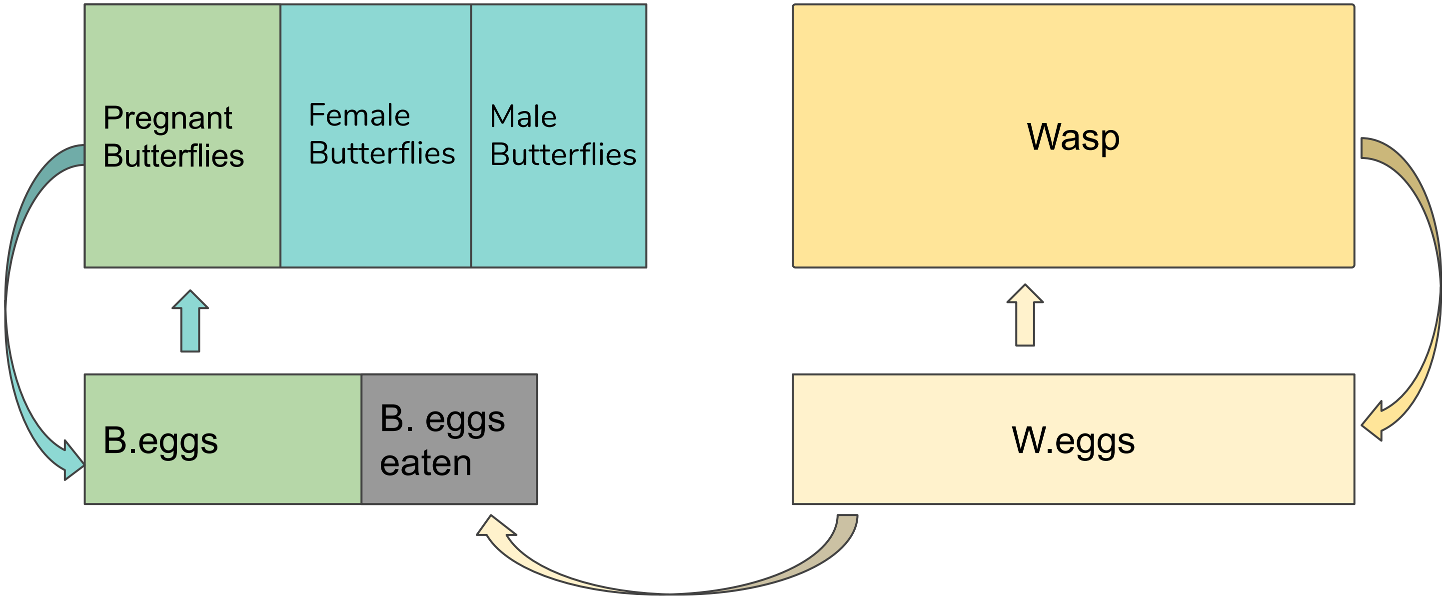
As the prompt stated:

“.....One question that arises is to determine the trade-offs and balance between the two competing interests. To do so develop a mathematical model for the interactions of the male and female P. brassicae as well as the parasitic wasps. What is the best balance for this system and what is likely to happen in the long run?”

As we interpreted it:

- The “competing forces” here refer to the trade-offs of releasing the anti-signal
- The interaction between male and female butterflies can be modelled with a constant, **l**
- The interaction between the wasp community and the butterfly community can be modelled with another constant **k**
- In order to predict the long run behaviour of these two populations, we vary our model according to varying values of l and k, and monitor the the change of population of either community

Stages of Modeling: Problem Breakdown



Stage 1: Use the Lotka–Volterra Equations: the Predator-Prey Model

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$

where

x is the number of prey (for example, rabbits);

y is the number of some predator (for example, foxes);

$\frac{dy}{dt}$ and $\frac{dx}{dt}$ represent the instantaneous growth rates of the two populations;

t represents time;

$\alpha, \beta, \gamma, \delta$ are positive real parameters describing the interaction of the two species.

Image Source: Wikipedia

Our approach:

1. Take x to be the number of butterflies, and y to be the number of wasps.
2. Determine $\boldsymbol{\alpha\,\beta\,\gamma\,\delta}$ by random/educated guesses.
3. Plot it on MatLab and see if it makes sense.

Shortcomings of this model

1. Too simple and we could make little modification to it
2. We don't account for pregnant butterflies and non-pregnant butterflies separately
3. What do $\boldsymbol{\alpha\,\beta\,\gamma\,\delta}$ even mean in our scenario!

Stage 2: Come up with our own model, that takes into account all the different interactions between the different species

After struggling to fit our model within the confines of the prey-predator model, we decided to analyze the situation step by step. So, we brainstormed all the different variables that could affect the situation, and first came up with smaller equations that made sense, and then combined them into one big equation.

The result?

$$\frac{dW(t)}{dt} = \lambda k W(t) B(t) - \beta W(t)$$

$$\frac{dB(t)}{dt} = (\mu P - \gamma) B(t) - \alpha k W(t) B(t)$$

λ= Average number of eggs laid by the wasps

μ=Average number of eggs laid by the female butterfly

k= Interaction rate between the butterfly population and the wasp population

β= Death rate of wasps

γ= Death rate of butterflies

P= Population ratio of the butterflies (female population: total population)

W(t)= Population of wasps at time t

B(t)= Population of butterflies at time t

Test of Models

- Since we developed a system of differential equations, it was relatively hard to do integration by ourselves.
- So we tested both models using MATLAB from the second stage and the third stage onwards.
- And the phase plane we plot out shows possible solutions that fit our model with different initial cases (i.e. different initial populations of wasps and butterfly population).

Testing Model for Stage 2

For the second stage, after we deliberately chose all the constants, the long run behaviour of wasps and butterflies reaches to a dynamic balance.

P = 0.6 f.pop : tot pop

μ = 200 eggs laid by female

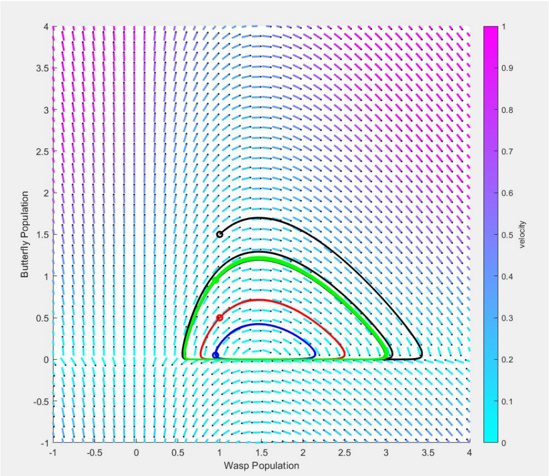
α = 60 eggs eaten by wasps

λ = 30 wasps eggs

β = 0.4 death rate of wasps

γ = 0.2 death rate of butterfly

k=0.4



Strange!
Because the butterfly population regrows from zero!

Stage 3: Refine the model

Third Stage: After we realized that we didn't take the interaction between male and female butterflies into account, we introduced a new control variable **l** to describe and quantify the behaviour within the butterfly species under the influence of anti-aphrodisiac. And also we changed the definition of **k:** from the interaction rate between two species to the interaction rate between wasps and pregnant butterflies.

$$\frac{dW(t)}{dt} = \lambda k W(t) B(t) - \beta W(t)$$

$$\frac{dB(t)}{dt} = l\mu P B(t)(1 - P) B(t) - \alpha k W(t) B(t) - \gamma B(t)$$

a= number of wasps attracted per female butterfly

b= number of wasp eggs laid per wasp

c= number of butterfly eggs destroyed per wasp egg

α= a*b*c. [This means that if we performed dimensional analysis on a, b and c, λ would be equal to the number of butterfly eggs eaten per female]

λ= a*b [If we do dimensional analysis on a and b, this would mean that α is the cost("overhead") of producing wasp eggs per female butterfly]

P = 0.6 is the ratio of female butterfly to the total number of butterflies. Therefore, this constant has no units.

μ= the average number of eggs laid per female butterfly.

K= the interaction rate between the pregnant butterflies and parasitic wasps. It is important to be noted that here k, stands only for the number of pregnant butterflies, and not the whole butterfly population. As was noted above and in the paper by Huigen et al, the parasitic wasps rides only on non-virgin female butterflies. And hence the only interaction that is of concern to us in this paper, is that between the wasps and the female butterfly.

l is the interaction rate between male and female butterfly, and we keep it variable also to see how the rate makes influence.

γ = 0.2 is the death rate of butterflies.

β = 0.4 is the death rate of wasps.

B(t) is the butterfly population at time t.

W(t) is the wasp population at time t.

Test of Model 3

For the third stage, we got a different pattern of our plot. There will be only two cases for the two populations: they either die out together or boost up together.

P = 0.6 f.pop : tot pop

μ = 200 eggs laid by female

α = 60 eggs eaten by wasps

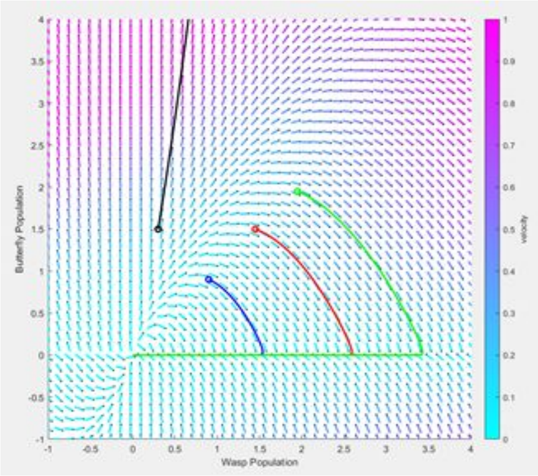
λ = 30 wasps eggs

β = 0.4 death rate of wasps

γ = 0.2 death rate of butterfly

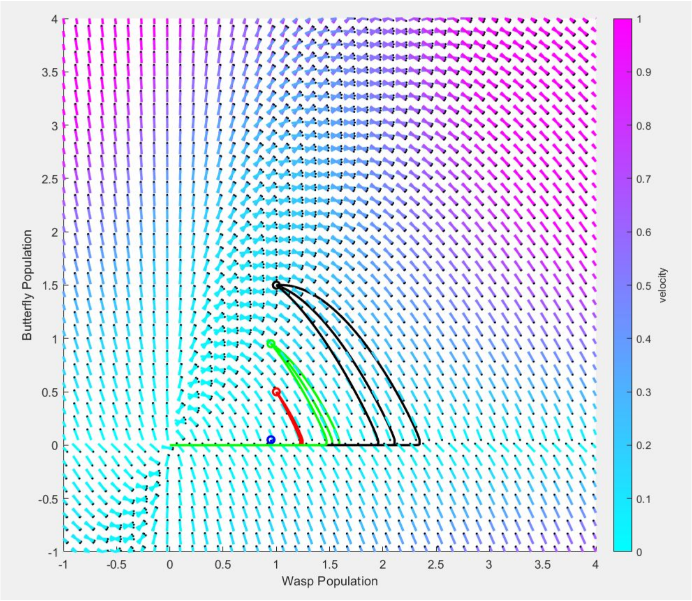
l=0.4

k=0.4

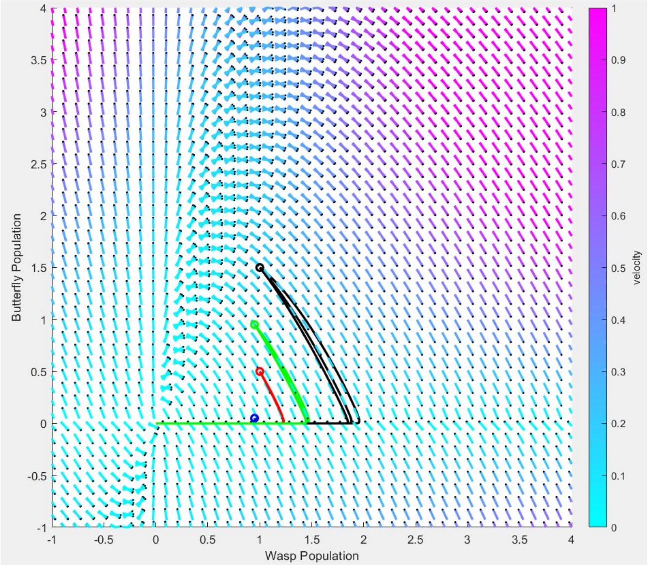


Not quite perfect!
Because most of the cases the two population just die out!

Sensitivity Analysis: Comparing the equations in two ways



Keeping **k** constant



Keeping **l** constant

Conclusion

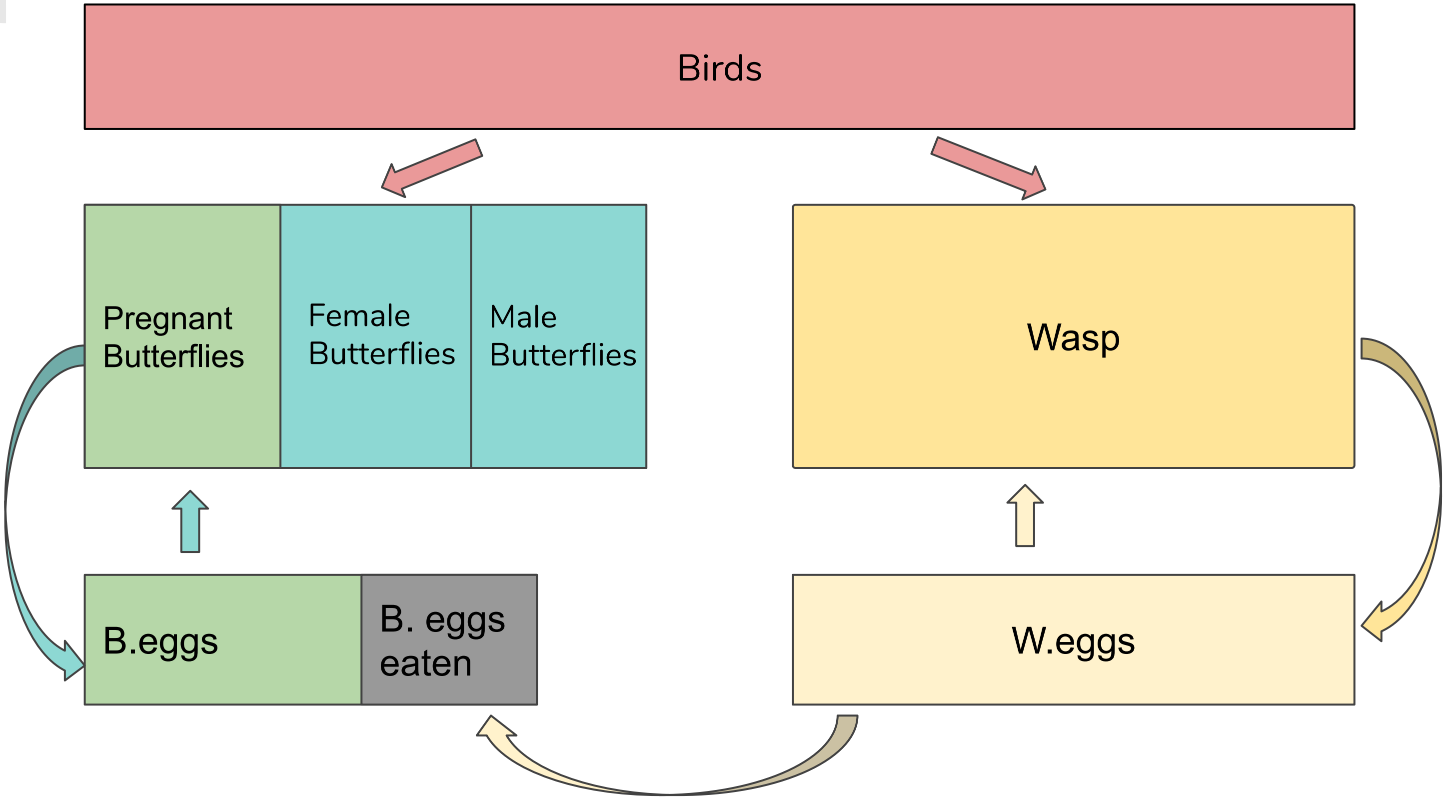
Based on the analysis of the plots from our differential equation systems, we come to conclusions that:

- According to our model, if we choose certain specific constants for $\alpha\beta\gamma\lambda$ and μ, the only possibility for the butterfly population to survive in the presence of the wasp population would be if we started out with a lot more butterflies than wasps.
- For a given ratio of butterflies and wasps, as the interaction rate between two species (i.e. k) increases, the two populations are more likely to die out; and as the interaction rate within butterfly species (i.e. l) increases, the two populations are more likely to grow together. And k and l always keep move in the same direction so that they balance each other in some way.
- The data is more sensitive to **l**, which is the interaction within the butterfly species, since in our model it is multiplied with a square term. Hence, to ensure long term sustainability of our environment, more females and males should mate, even if it means releasing more anti-signal.

Improvement

- **Problem 1**: Under certain initial conditions, butterfly and wasp population grows exponentially, which is not reasonable
- **Improvement 1**: Incorporate carrying capacity to the equation to model a more realistic scenario
- **Problem 2**: Under certain other initial conditions, both populations always eventually die out
- **Improvement 2:** Pick more realistic constants for the birth rate of butterfly and the death rate of wasps
- **Problem 3**: The model assumes that wasps only lay eggs when they sense butterflies, which in reality is not the case. Wasps also have other sources for food.
- **Improvement 3**: Add another term in the W(t) equation to have the wasp population's growth rate to be independent of the butterfly population.

Additional Issue 1: Another Predator



Additional Issue

Problem Statement: Assume that in addition to the system of butterflies, and wasps, we have an additional predator: the bird, which preys on both butterflies and wasps

Additional Issue 1

Assumption:
The only food source for the bird are the wasps and the butterflies.
The only food source for the wasp is the butterfly. The butterfly has abundant food.

Equations

$$\frac{d\boldsymbol{W}(t)}{dt} = \lambda k \boldsymbol{W}(t)\boldsymbol{B}(t) - \beta \boldsymbol{W}(t) \quad (1)$$

$$\frac{d\boldsymbol{B}(t)}{dt} = l\mu P\boldsymbol{B}(t)(1-P)\boldsymbol{B}(t) - \alpha k \boldsymbol{W}(t)\boldsymbol{B}(t) - \gamma \boldsymbol{B}(t) \quad (2)$$

$$\frac{d\boldsymbol{R}(t)}{dt} = \delta_1 \boldsymbol{R}(t)\boldsymbol{B}(t) + \delta_2 \boldsymbol{R}(t)\boldsymbol{W}(t) - \sigma R(t) \quad (3)$$

- R(t) here is the bird population with respect to time
- ρ1 represents the interaction rate between the bird population and the butterfly population
- ρ2 represents the interaction rate between the bird population and the wasp population

References

[1] “Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps, “ Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros.Behavioral Ecology. Volume 21, Issue 3, May-June 2010, Pages 470–478, 11 February 2010. https://doi.org/10.1093/beheco/arq007

[2] https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations

Thank you!