Problem C

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CHEMICAL ESPIONAGE

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BACKGROUND

- Once a female butterfly mates with a male, the male uses anti-aphrodisiacs to deter other males.
  - This allows the female to find a better place to lay her eggs
- These anti-aphrodisiacs can be detected by two different types of parasitic wasps.
  - The wasp hitchhikes to where the butterfly lays her eggs and lays its eggs there as well.
- The butterflies must find a balance between deterring other males and preventing the wasps from hitchhiking.

ASSUMPTIONS

- The environment does not limit butterfly population
- The butterflies are the only food source for the wasps
- Butterflies are modeled as a single species, rather than as male and female separately
- Males can control the *amount* of anti-aphrodisiacs utilized
- Initially assumed that wasps don't interact. This assumption changed after analyzing our model

OVERALL APPROACH

- Decided to choose basic format to explore & modify
- We chose a Lotka-Volterra population model

- Basic Prey Response (Butterflies):

$$\frac{dP}{dt} = \alpha P - \beta PW$$

- Basic Predator Response (Wasps):

$$\frac{dW}{dt} = -\gamma W + \delta PW$$

OVERALL APPROACH

- Decided to choose basic format to explore & modify
- We chose a Lotka-Volterra population model

- Modified Prey Response (Butterflies):

$$\frac{dP}{dt} = \alpha P - \beta_1 P W_1 - \beta_2 P W_2$$

- Modified Predator Response (Wasps):

$$\frac{dW_1}{dt} = -\varphi W_1 + \delta P W_1 - A W_1 W_2$$

$$\frac{dW_2}{dt} = -\gamma W_2 + \lambda P W_1 - B W_1 W_2$$

ANALYSIS

- We solved our model for equilibrium solutions, three of which occur when any one population is extinct. The fourth solution was the focus of our analysis.

$$P = \frac{AB\alpha + A\beta_1\gamma + B\beta_2\phi}{A\beta_1\lambda + BA\beta_2\delta},\ W_1 = \frac{A\alpha\lambda - \beta_2\delta\gamma + \beta_2\lambda\phi}{A\beta_1\lambda + B\beta_2\delta},\ W_2 = \frac{B\alpha\delta + \beta_1\delta\gamma - \beta_1\lambda\phi}{A\beta_1\lambda + B\beta_2\delta}$$

- $AB\alpha$ term shows increase in butterfly population
- $\frac{\gamma}{\lambda}$ and $\frac{\phi}{\delta}$ also show increase in butterfly population
  - γ, ϕ are the natural decay rates of wasp populations
  - λ, δ are the interaction parameters between wasps and butterflies

$$\alpha \propto \kappa^n$$

$$\delta, \lambda, \beta_1, \beta_2 \propto \kappa^m$$

$$n < m$$

ADDITIONAL ISSUE

- We were asked to consider the impact of a female butterfly being able to detect a male's butterfly's propensity to use the anti-aphrodisiac

$$\alpha \propto \kappa^n$$

$$\delta, \lambda, \beta_1, \beta_2 \propto \kappa^m$$

$$n < m$$

- For low values of κ, α is more positively impacted (Butterfly population benefits)
- Some threshold value for propensity to use aphrodisiac represented as κ=1, above which usage benefits wasp population
- Female butterflies should select males with lower (but still nonzero) propensity: $0 < \kappa < 1$

FUTURE IMPROVEMENTS

- Generating expressions for the parameters
  - In terms of the amount of anti-aphrodisiac used by the butterfly population
  - Would enable us to investigate the effects of the butterflies using the anti-aphrodisiac more or less frequently
- Using Type II and Type III functional response in the modeling of the wasps
  - More accurately represent the real-world predator-prey situation

CONCLUSION

- High use of anti-aphrodisiac negatively affects butterfly population
- Butterfly population may benefit from use of anti-aphrodisiac in small amounts