Chemical Espionage - Problem C

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Problem C Specification

- Male *P. brassicae* use anti-aphrodisiacs to mask a female butterfly from other males.
- Parasitic wasps can detect the anti-aphrodisiacs, follow the female butterfly, and lay their own eggs in *P. brassicae* eggs.
- Develop a mathematical model for the interactions of the male and female *P. brassicae* as well as parasitic wasps.

Assumptions

- Male-to-female ratio in the butterfly and wasp population is constant.
- Probability of being targeted by wasps is the same for every female *P. brassicae.*
- Parasitizing any non-mated female butterfly has no effect on butterfly population size.

Assumptions

- Every female has an equal number of potential “suitors”.
- There are no mutations or evolution in the populations studied.
- Growth and death factors are constant for both populations.
- Wasps parasitize only on P. brassicae population.

Competing interests

- Male *P. brassicae* using more anti-aphrodisiac means the eggs are better hidden and more of them are fertilized.

- On the other hand, more anti-aphrodisiac attracts more parasitic wasps.

Lotka-Volterra Model of Competition

Logistic model:

$$\frac{\partial X}{\partial t} = r_x * X * (1 - \frac{X}{K_x})$$

Predator-prey model:

$$\begin{cases} \dfrac{\partial X}{\partial t} = r_x * X * (1 - \dfrac{X - \alpha_{xy} * Y}{K_x}) \\ \dfrac{\partial Y}{\partial t} = r_y * Y * (1 - \dfrac{Y + \alpha_{yx} * X}{K_y}) \end{cases}$$

Variables:

- X, Y – populations.
- r_x, r_y – growth factors for populations X and Y.
- K_x, K_y – carrying capacities for populations X and Y.
- α_{xy} – competition coefficient for population X.
- α_{yx} – competition coefficient for population Y.

Differential Equations – Male and Female Interactions

$$\frac{\partial P_1}{\partial t} = P_1 * \left(1 - \frac{P_1 + \alpha_{12} * M_{aa} * P_2}{K_1}\right) * mf * M_{aa} - df_1 * P_1$$

- P_1– population of P. brassicae. r_1– growth factor of P. brassicae.
- K_1– carrying capacity for P. brassicae.
- α_{12} – competition coefficient, shows how P. brassicae population's resources are affected by wasps .
- mf – percentage of mated female P. brassicae.
- M_{aa} – amount of anti-aphrodisiac released by each male P. brassicae.
- df_1– natural death factors for P. brassicae.
- t – time.

Differential equations – Wasp population

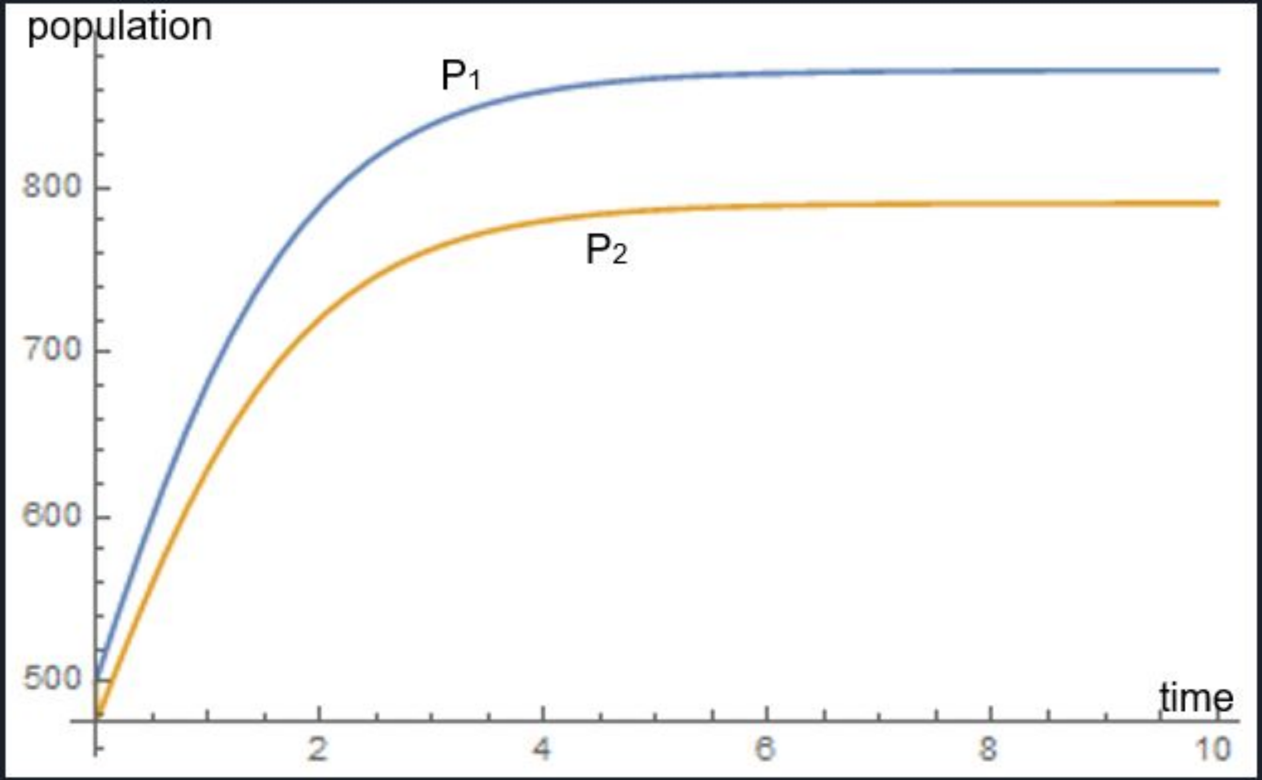
$$\frac{\partial P_2}{\partial t} = P_2 * \left(1 - \frac{P_2 - \alpha_{21} * M_{aa} * P_1 + \alpha_{23} * P_3}{K_2}\right) * mf * P_1 - df_2 * P_2$$

- P_2, P_w – population of Trichogramma brassicae.
- r_2 – growth factor of T. population.
- K_2 – carrying capacity for T. brassicae population.
- α_{21} – competition coefficient, shows how T. brassicae resources are affected by P. brassicae.
- mf – percentage of mated female P. brassicae.
- M_{aa} – amount of anti-aphrodisiac released by each male P. brassicae.
- df_2 – natural death factors T. brassicae population respectively.
- t – time

Balance of Butterfly and Wasp Population

Population equilibrium occurs in the model when neither of the population levels is changing, i.e. when both of the derivatives are equal to 0.

Long Run and Visualization



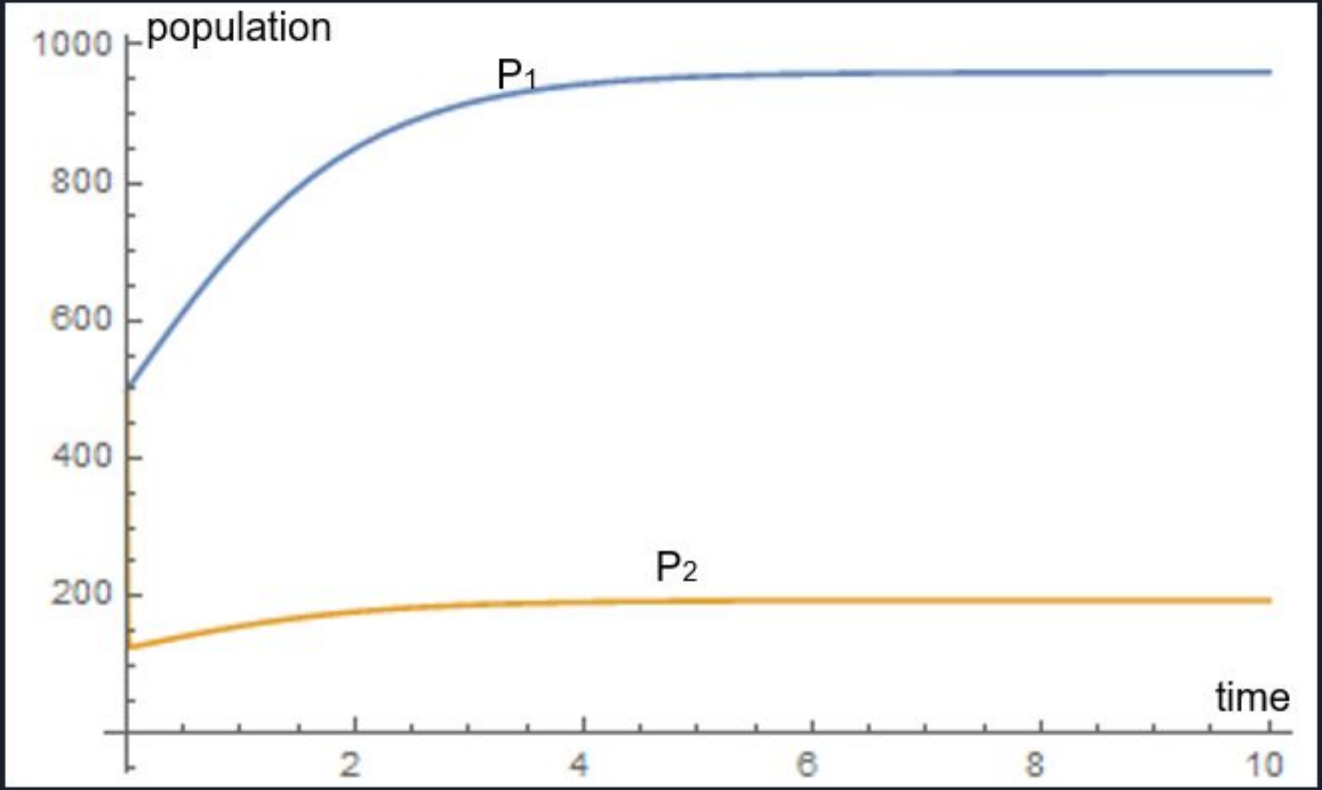
$$K_1 = 1000, \alpha_{12} = 0.1$$

$$K_2 = 50, \alpha_{21} = 0.9$$

$$\frac{\partial P_1}{\partial t} = P_1 * \left(1 - \frac{P_1 + 0.15 * P_2}{1000}\right) - 0.01 * P_1$$

$$\frac{\partial P_2}{\partial t} = 0.13 * \left(1 - \frac{P_2 - 0.85 * P_1}{50}\right) - 0.01 * P_2$$

Long Run and Visualization



$$K_1 = 1000, \alpha_{12} = 0.1$$

$$K_2 = 50, \alpha_{21} = 0.1$$

$$\frac{\partial P_1}{\partial t} = P_1 * \left(1 - \frac{P_1 + 0.15 * P_2}{1000}\right) - 0.01 * P_1$$

$$\frac{\partial P_2}{\partial t} = 0.13 * \left(1 - \frac{P_2 - 0.1 * P_1}{50}\right) - 0.01 * P_2$$

Conclusions

- The main trade-off between the two populations is the amount of anti-aphrodisiac released by male P. brassicae.
- In the long run, both populations will eventually stabilize at certain levels.

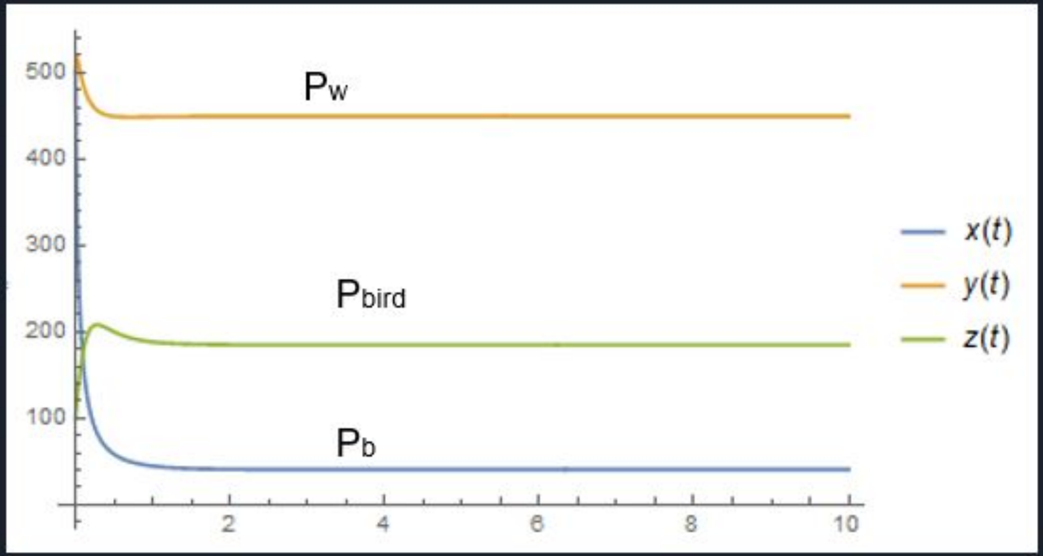
Additional Problem

- Suppose you are asked to add an animal that is a predator of both the butterflies and the wasps, a bird for example. How would you change your model to accommodate this new situation?

- Lotka-Volterra model of competition can be changed to account for more than two species by adding another competition coefficient.

$$\begin{cases} \dfrac{\partial P_1}{\partial t} = P_1 * \left(1 - \dfrac{P_1 + \alpha_{12} * M_{aa} * P_2 + \alpha_{13} * P_3}{K_1}\right) * mf * M_{aa} - df_1 * P_1 & \text{Butterflies} \\ \dfrac{\partial P_2}{\partial t} = P_2 * \left(1 - \dfrac{P_2 - \alpha_{21} * M_{aa} * P_1 + \alpha_{23} * P_3}{K_2}\right) * mf * P_b - df_2 * P_2 & \text{Wasps} \\ \dfrac{\partial P_3}{\partial t} = P_3 * \left(1 - \dfrac{P_3 - \alpha_{31} * P_1 - \alpha_{32} * P_2}{K_3}\right) - df_3 * P_3 & \text{Birds} \end{cases}$$

Additional Problem



$K_1 = 100, \alpha_{12} = 0.05, \alpha_{13} = 0.2$ Butterflies

$K_2 = 500, \alpha_{21} = 0.15, \alpha_{23} = 0.3$ Wasps

$K_3 = 30, \alpha_{31} = 0.5, \alpha_{32} = 0.3$ Birds

Thank you for your attention!

Sources:

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- Martinus E. Huigens, et al. : Hitch-Hiking Parasitic Wasp Learns to Exploit Butterfly Antiaphrodisiac. https://www.jstor.org/stable/40254755
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