SCUDEM IV 2019: Chemical Espionage (Problem C)

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Problem:

Develop a mathematical model to illustrate the interactions of male and female *Pieris Brassicae* with a parasitic wasp species. What is the best balance for this system?

Assumptions

- *P. Brassicae* has an infinite life span
- Wasp hijacks are the only limiting factor
- Closed population: no migration or emigration
- 50% of the population is male
- Only competing males are affected by another male's anti-aphrodisiac
- A wasp that detects anti-aphrodisiacs always succeeds in hijacking

Equations

$$\frac{dP}{dt} = P(R - LW_p)$$

$$\frac{dW_p}{dt} = LPW_p - KW_p$$

Derivation Process

- The growth rate of the butterflies is proportional to the population, the constant of proportionality would stand for the chance of detection of the aphrodisiac by male butterflies:

$$\frac{dP}{dt} = RP$$

- We then incorporated the effect of the Wasp population, subtracting the rate of eggs being hijacked, which we claimed was proportional to both the butterfly and wasp population.

$$\frac{dP}{dt} = P\left(R - LW_p\right)$$

Derivation process cont.

- We realized that the wasp population was also variable, and in our model was depended on the wasp population itself times the butterfly population and a constant describing the chance of a wasp detecting anti-aphrodisiac:

$$\frac{dW_p}{dt} = LPW_p$$

- We then incorporated a death rate for the wasps to balance the system and give the butterflies a chance

$$\frac{dW_p}{dt} = LPW_p - KW_p$$

Solving the Differential Equations:

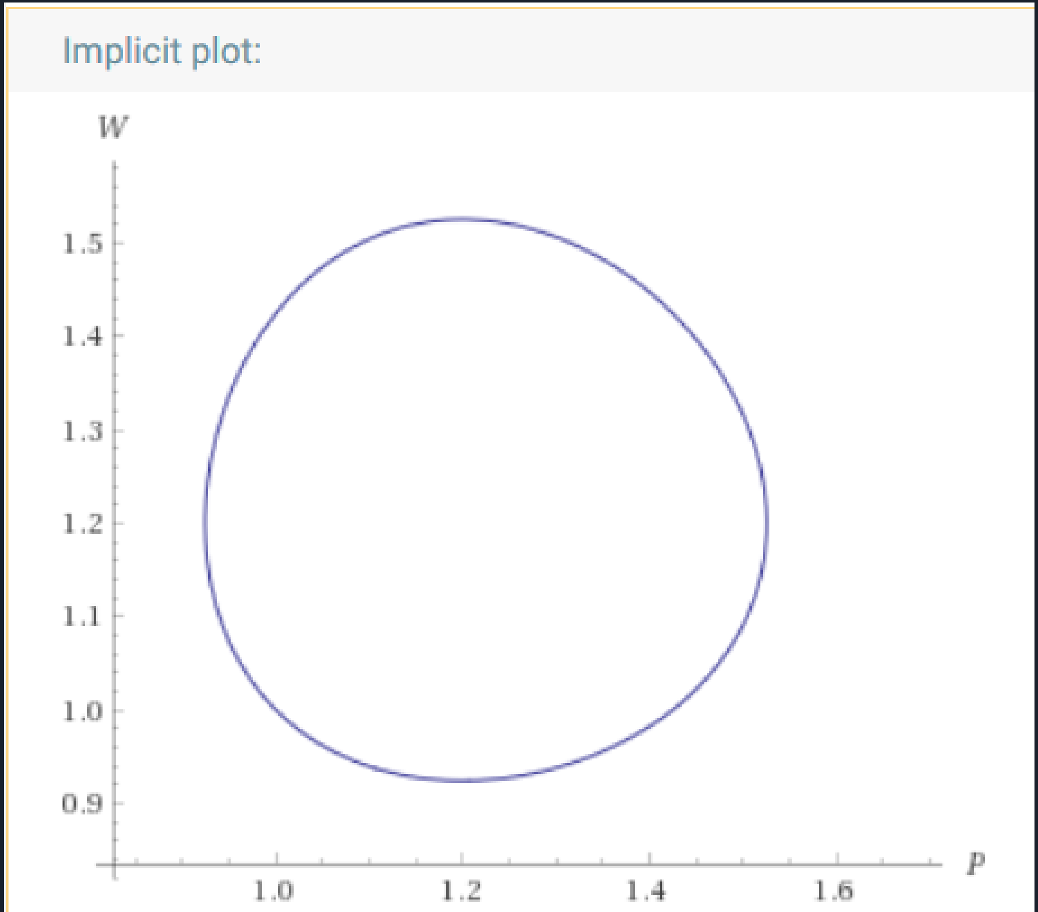
- We realized that our model represented Lotka-Volterra equations, or predator prey equations. These types of differential equations are difficult to solve in terms of simple functions. Lotka-Volterra equations dictate a method where we can divide the differential equations by each other to eliminate time:

$$\frac{dP}{dW_p} = \frac{P\left(R - LW_p\right)}{LPW_p - KW_p}$$

- Separating and integrating results in an implicit expression:

$$LP - Kln(P) = Rln\left(W_p\right) - LW_p + C$$

Graph:



Equilibrium

- By setting dP/dT and dW_p/dt =0, we can find the point of equilibrium where the population of wasps and the population of butterflies no longer change.

$$\frac{dP}{dt} = RP - lPW_p = 0$$

$$\frac{dW_p}{dt} = LPW_p - KW_p = 0$$

- giving two solutions:

$$P = 0\,, \qquad W_p = 0$$

$$P = \frac{K}{L},\ W_p = \frac{R}{L}$$

Limitations

- Doesn’t account for resource limitations
- No death rate for butterflies
- No migration, closed system is somewhat unrealistic
- 100% chance of fertilization if wasp is not involved
- Even number of males and females; not always monogamous

Additional Issue

- We chose Problem C #1:
- Introduce another predator to the model which would effect the populations of both the wasps and the butterflies: How would you change your model to accommodate this new situation? How would this effect the model?
- New differential equation in terms of population of butterflies and wasps and a constant of proportionality describing both the chance of predation and resultant growth rate towards production of a new predator
- Introduce a death rate to the birds, Bird population times a constant of proportionality

$$\frac{dB}{dt} = QMB(W_p + P) - SB$$

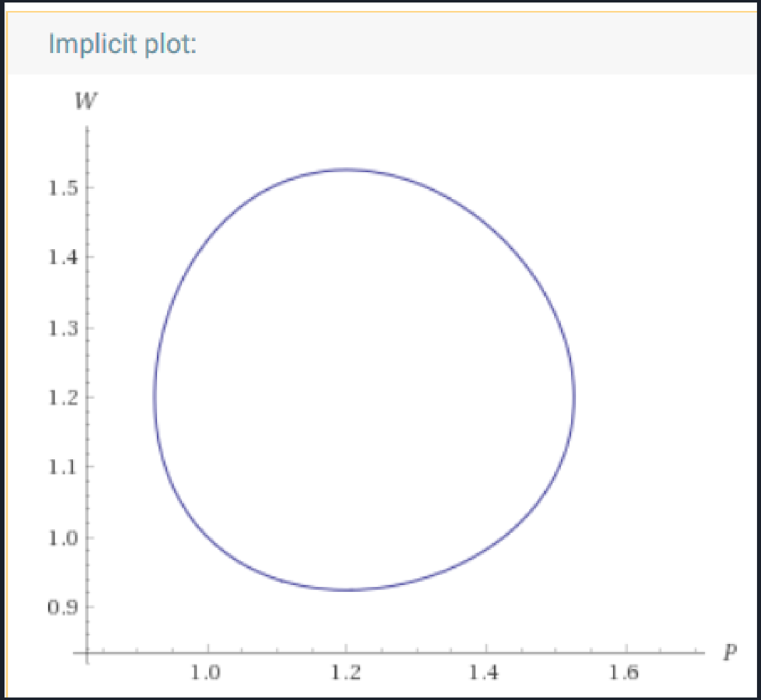
Additional issue cont.

- We would adjust our differential equations for the other populations:

$$\frac{dP}{dt} = P(R - LW_p - MB)$$

$$\frac{dW_p}{dt} = W_p(LP - K - MB)$$

Graph characteristics:



Works Cited:

- “Lotka–Volterra Equations.” Wikipedia. Wikimedia Foundation, November 7, 2019. https://en.wikipedia.org/wiki/Lotka–Volterra_equations.