CHEMICAL ESPIONAGE

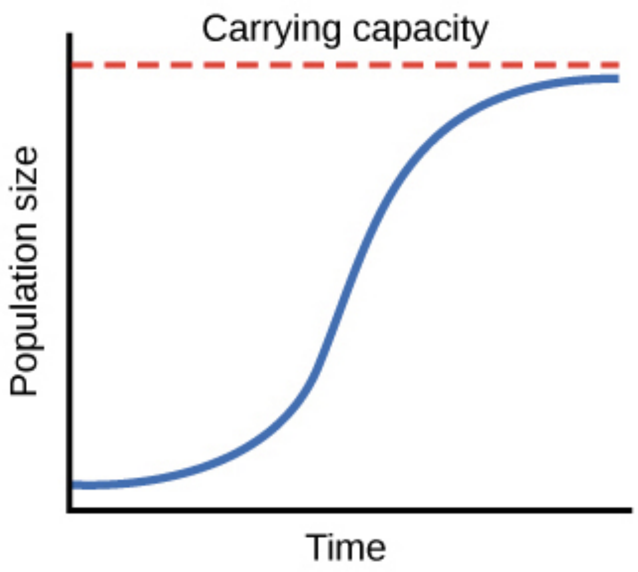
PASCAL, FRED, SYLVESTER

BACKGROUND AND THOUGHT PROCESS

- Female butterfly release chemical signals that attract males.
- Males releases anti-aphrodisiacs which mask or dissuade other males.
- However, wasps can follow the male’s scent and lay their own eggs in the butterflies’ eggs, killing the butterfly offspring in the process.
- We adopted different models to represent this scenario, mostly focusing 3 parameters the number of butterflies, the number of wasps and the strength or range of male chemical signal.

SECOND APPROACH AND ASSUMPTIONS

- Considering a logistic population growth that levels off at carrying capacity – maximum population size that the environment sustain.
- Population is governed by equation with K that represents the carrying capacity.
- We assumed that the signal strength is a constant.



$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

FIRST MODEL

- $\frac{dw}{dt} = Z1 * \mathrm{a} * \mathrm{b} * \mathrm{w}$
- $\frac{dn}{dt} = Z2 * (a * n - a * b * w)$
- **w**: number of wasps; **n**: number of butterflies; **Z1**: birth rate of wasps; **Z2**: birth rate of butterflies;
- **a**: probability that a male butterfly is within range of a female butterfly signal.
- **b**: probability that a wasp is within range of a male butterfly signals.
- Graphing Calculator

(1) $\frac{dw}{dt} = z_1 \times a \times bw$

$-e^{-t}+1$

(2) $\frac{dn}{dt} = z_2(an - ab\ w)$

Step (1)

Step (3) : carrying capacity

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(1) $\frac{dw}{dt} = z_1 abw$

$\int \frac{1}{w} dw = \int z_1 ab\ dt$.

$\ln|w| = z_1 abt + c$

$\boxed{w = Ae^{z_1 abt}}$

$p(n=0) = 0$

$p(n=\infty) = 1$

(2) $\frac{dn}{dt} = z_2\left(an - abAe^{z_1 abt}\right)$

$\frac{dn}{dt} = z_2 an - z_2 abAe^{z_1 abt}$

$\frac{dn}{dt} - (z_2 a)n = -z_2 abAe^{z_1 abt}$

$C + e^{-z_2 at} . n = \int \left(-e^{-z_2 at}\right) \left(z_2 abAe^{z_1 abt}\right) dt$

$\mu = e^{\int -z_2 a\, d}$

$= e^{-z_2 at}$.

$$-z_2 abA \int e^{-z_2 at} e^{z_1 abt}\, dt.$$

$$-z_2 abA \int e^{(-z_2 a + z_1 ab)t}\, dt.$$

$$-z_2 abA\left(-\frac{e^{(-z_2 a + z_1 ab)t}}{z_2 a + z_1 ab}\right)$$

$$\mu = e^{\int -z_2 a\, dt}.$$
$$= e^{-z_2 at}.$$

$$\left(\frac{z_2 bA}{z_2 + z_1 b}\right) e^{(-z_2 a + z_1 ab)t} = c + n e^{-z_2 at}$$

$$\boxed{\left(\frac{z_2 bA}{z_2 + z_1 b}\right) e^{z_1 abt} + c e^{z_2 at} = n}$$

$t = 0$

$t = 0$

$$|c| > \frac{z_2 bA}{z_2 + z_1 b}.$$

if $c = \dfrac{-z_2 bA}{z_2 + z_1 b}$

$+ \; c < 0$

SECOND MODEL

- $\frac{dw}{dt} = r1 * w * (\frac{a*\gamma*n+k1-\omega}{a*\gamma*n+k1})$
- $\frac{dn}{dt} = r2 * n * \left(\frac{k2-n}{k2}\right) - b * w$
- $\gamma = c * n$
- **w** : number of wasps; **n** : number of butterflies males; **γ** : male signal strength

SOLUTION TO SECOND MODEL

- We used coupled system of differential equations to solve above equations.
- The independent variable is t and the two dependent variables are w and n.

$\frac{dw}{dt} = r_2 w\left(\frac{a}{a} \quad \frac{+K_1 - W}{+h}\right)$

$= r_2 w\left(\frac{K_2 - n}{K_2}\right) - abnw$

$= an$

② $w = r_2 n\left(\frac{K_2-n}{K_2 b}\right) - \frac{dn}{dt}\cdot\frac{1}{b}$

$$\frac{d\left(r_2 n\left(\frac{K_2-n}{K_2 b}\right) - \frac{dn}{dt}\cdot\frac{1}{b}\right)}{dt} = r_1\left(r_2 n\left(\frac{K_2-n}{K_2 b}\right) - \frac{dn}{dt}\cdot\frac{1}{b}\right)\left(\frac{(acn^2+K_1) - \left(r_2 n\left(\frac{K_2-n}{K_2 b}\right) - \frac{dn}{dt}\cdot\frac{1}{b}\right)}{acn^2+K_1}\right)$$

$$\frac{d}{dt}\left(\frac{r_2 n}{b}\right) - \frac{d}{dt}\left(\frac{r_2 n^2}{K_2 b}\right) - \frac{d^2}{dt^2}\left(\frac{n}{b}\right) = \left(\frac{r_1 r_2 n K_2 - r_1 r_2 n^2}{K_2 b} - \frac{d}{dt}\left(\frac{n r_1}{b}\right)\right)\left(1 - \frac{\frac{r_1 r_2 n K_2 - r_1 r_2 n^2}{K_2 b} + \frac{d}{dt}\left(\frac{r_1 n}{b}\right)}{r_1(acn^2+K_1)}\right)$$

$$\frac{d}{dt}(An) - \frac{d}{dt}(Bn^2) - \frac{d^2}{dt^2}(nC) = \left(D(nK_2 - n^2) - \frac{d}{dt}(En)\right)\left(1 - \frac{D(nK_2-n^2) + \frac{d}{dt}(En)}{r_1(acn^2+K_1)}\right)$$

$$\boxed{\frac{d}{dt}(An) - \frac{d}{dt}(Bn^2) - \frac{d^2}{dt^2}(Cn) = -\frac{\left(D(nK_2-n^2)\right)^2 - \left(\frac{d}{dt}(En)\right)^2}{r_1(acn^2+K_1)} + D(nK_2-n^2) - \frac{d}{dt}(En)}$$