Drake University Team 3

Kaela Newman, Justin Brutger, David Atri Schuller

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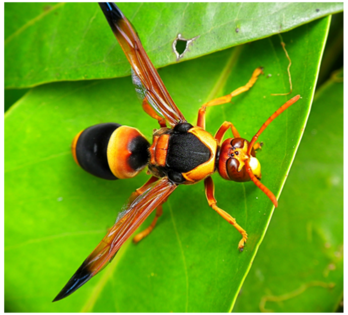


Problem C - Chemical Espionage

The Problem: Chemical Espionage

- Butterfly and wasp populations compete
- Female butterflies release chemical signals to attract males for mating
- In response to the female chemical signals, male butterflies release anti-aphrodisiacs to deter other males from grouping up on individual females, leading to more efficient breeding.
- Wasps detect the chemical signals and lay “parasite eggs”.
- Anti-aphrodisiacs allows butterflies to mate efficiently, but also attracts wasps that kill their eggs.

Assumptions

- The butterfly population is essential for wasp survival.
  - This means the natural death rate for wasps is greater than the natural growth rate.
- The amount of anti-aphrodisiacs present is proportional to the interactions between male and female butterflies.
- There is some standard birth rate of butterfly eggs
- The rate of viable eggs laid by butterflies is proportional to the amount of anti-aphrodisiacs present and the interaction between male and female butterflies
- Butterflies eggs are taken over by parasitic wasps at a rate proportional to the amount of anti-aphrodisiacs and the amount of wasps.
- Each butterfly egg compromised results in one wasp egg being laid.
- Butterfly Eggs have equal likelihood of being male or female

The Model: Beginnings

- Chemicals are released by female butterflies
- Chem(t) = c*F(t)
- Male butterflies release anti-aphrodisiacs to deter other males:
- AA(t) = Chem(t) *M(t)
- Butterfly birth rate is proportional to the anti-aphrodisiacs and the interaction of male and female butterflies:
  - ‘Raw Births’ = c * AA(t) * F(t) * M(t)
- Wasp parasitism is proportional to the amount of aphrodisiacs and the amount of wasps:
  - ‘Parasitism’ = c * AA(t) * W(t)
- Each population has a death rate proportional to said population.
- Wasps have a natural growth rate proportional to their total population.

The Model: Final

- With all aspects of growth and death combined:
- W’(t) = b*W(t) + c*M(t)*F(t)*W(t) - g*W(t)
- M’(t) = a*c*F(t)^2*M(t)^2/2 - c*M(t)*F(t)*W(t) /2- e*M(t)
- F’(t) = a*c*F(t)^2*M(t)^2/2 - c*M(t)*F(t)*W(t) /2- d*F(t)
  - All constants are positive, while g is greater than b as per our assumptions.
  - a = coefficient of eggs laid, c = anti-aphrodisiac effectiveness coefficient
  - g,e,d = respective death rates

Find Equilibria

- To find the equilibria we set each derivative equal to 0
  - 0 = b*W(t) + c*M(t)*F(t)*W(t) - g*W(t)
  - 0= a*c*F(t)^2*M(t)^2/2 - e*M(t)
  - 0 = a*c*F(t)^2*M(t)^2/2 - d*F(t)

- We found three realistic equilibrium solutions:

$$(0, 0, 0)$$

$$\left(0, \frac{2d}{ca\left(\frac{2d^2}{cae}\right)^{\frac{2}{3}}}, \sqrt[3]{\frac{2d^2}{cae}}\right)$$

$$\left(\frac{g-b}{e}\sqrt{\frac{de}{c(g-b)}}, \frac{e}{c}\sqrt{\frac{c(g-b)}{de}}, \frac{a(g-b)}{c} - 2\sqrt{\frac{de}{c(g-b)}}\right)$$

Find Stability of Equilibria

- Compute Jacobian Matrix Eigenvalues
  - Obtain 3 eigenvalues for each equilibrium solution
  - Equilibrium solution (W,F,M) is stable if all corresponding eigenvalues are negative

$$(0,0,0)$$

$$\lambda_1 = -e$$
$$\lambda_2 = -d$$
$$\lambda_3 = b - g$$

ALWAYS STABLE

$$\left(0, \frac{2d}{ca\left(\frac{2d^2}{cae}\right)^{\frac{2}{3}}}, \sqrt[3]{\frac{2d^2}{cae}}\right)$$

$$\lambda_1 = \frac{e+d}{2} + \frac{1}{2}\sqrt{e^2 + d^2 + 14ed}$$
$$\lambda_2 = \frac{e+d}{2} - \frac{1}{2}\sqrt{e^2 + d^2 + 14ed}$$
$$\lambda_3 = (b-g) + c^3\sqrt{\frac{4de}{ca^2}}$$

ALWAYS UNSTABLE

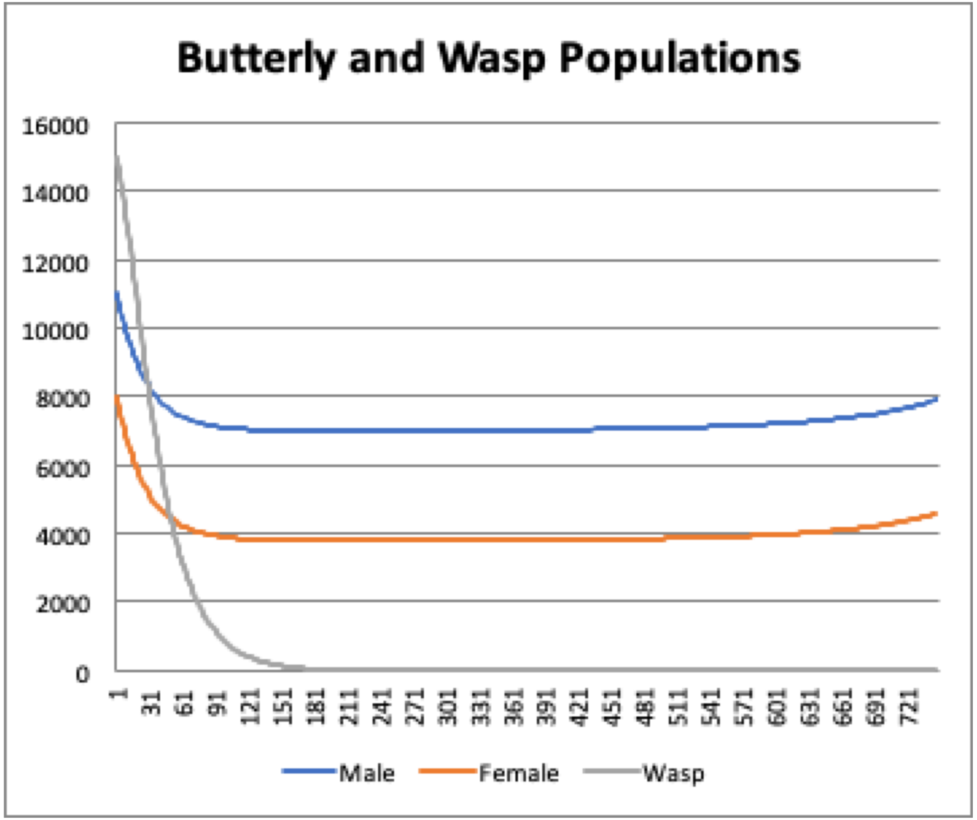
$$\left(\frac{g-b}{e}\sqrt{\frac{de}{c(g-b)}}, \frac{e}{c}\sqrt{\frac{c(g-b)}{de}}, \frac{a(g-b)}{c} - 2\sqrt{\frac{de}{c(g-b)}}\right)$$

Complicated cubic equation. Found eigenvalues numerically for varying parameters

STABILITY UNKNOWN

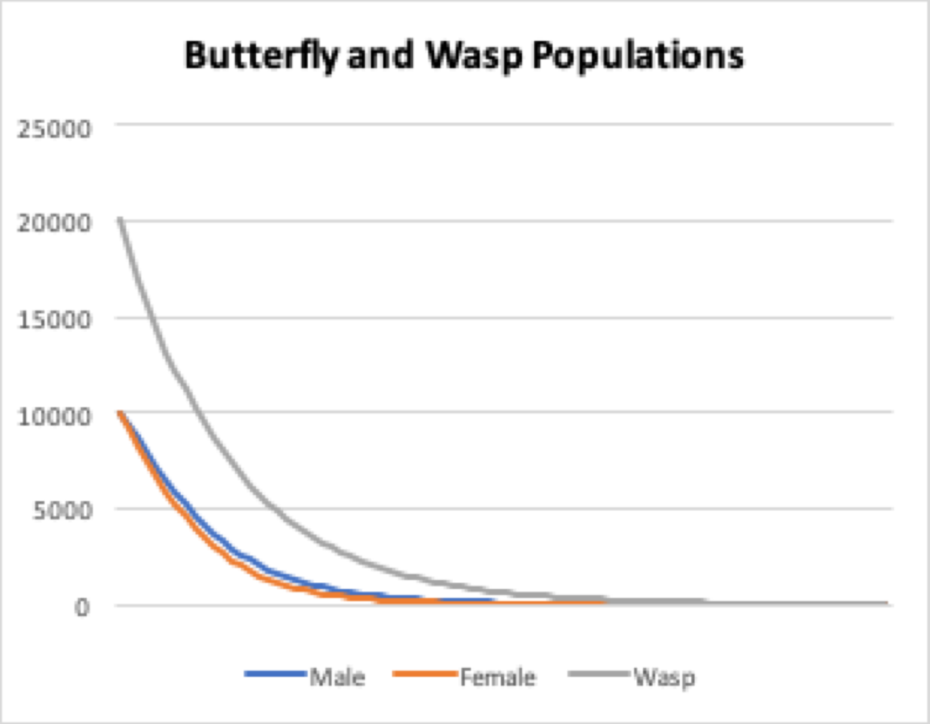
Find suitable parameters: Butterfly Survival

- Starting populations:
  - Male: 11,000
  - Female: 8,000
  - Wasp: 15,000
- Parameters:
  - a: 0.0001 (butterfly birth rate)
  - b: 0.04 (wasp growth)
  - c: 4e-10 (anti aphrodisiacs)
  - d: 0.0037 (female death)
  - e: 0.002 (male death)
  - g: 0.085 (wasp death)
- End Behavior
  - Wasps go to extinction
  - Butterfly population recovers
  - Approaches second equilibrium briefly
    - W(t) = 0
    - F(t) = 3,781
    - M(t) = 6,994

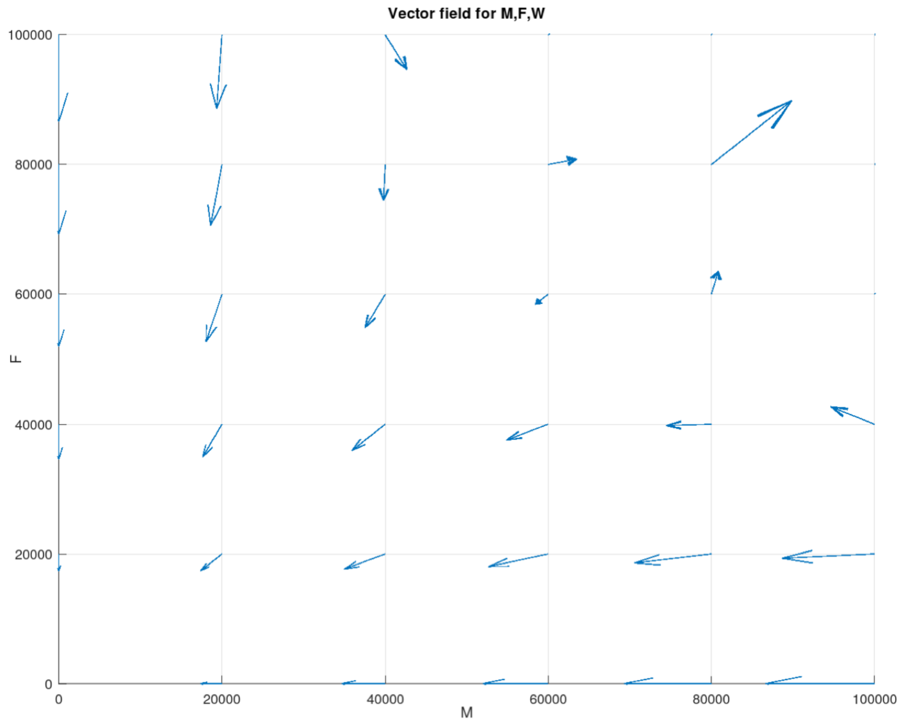


Find suitable parameters: Extinction Scenario

- Starting populations:
  - Male: 10,000
  - Female: 10,000
  - Wasp: 7,500
- Parameters:
  - a: 0.8 (butterfly birth rate)
  - b: 0.11 (wasp growth)
  - c: 1.1E-13(anti aphrodisiacs)
  - d: 0.13 (female death)
  - e: 0.11 (male death)
  - g: 0.19 (wasp death)
- Behavior
  - Due to a lack of high butterfly population, all three species will die out



Characterize model behavior



Additional Issues: Part 1

Add an animal that is a predator to both the butterflies and wasps.

We would change our model to add a new death term for each population that is proportional to the interactions between the predator and the population. We also would need to add an equation for our new predator.

- $W'(t) = b*W(t) + c*M(t)*F(t)*W(t) - g*W(t) - h*W(t)*P(t)$
- $M'(t) = a*c*F(t)^2*M(t)^2/2 - c*M(t)*F(t)*W(t) /2- e*M(t) - i*M(t)*P(t)$
- $F'(t) = a*c*F(t)^2*M(t)^2/2 - c*M(t)*F(t)*W(t) /2- d*F(t) - i*F(t)*P(t)$
- $P'(t) = j*W(t)*P(t) + k*P(t)(M(t) + F(t)) - m*P(t)$

The predator experiences a growth rate proportional to its interactions with each species but not necessarily in a 1:1 ratio as the death rate experienced by the preys. The predator also has its own base death rate, m.

Additional Issues: Part 2

The female butterfly can detect a male butterfly's propensity to use anti-aphrodisiacs prior to mating. What should her strategy be in choosing a mate?

In our model, butterflies have a higher likelihood of long-term survival if there are less anti-aphrodisiacs present. The negative impact anti-aphrodisiacs have on the eggs due to wasp consumption outweighs the benefits of optimized placement.

The female butterfly should choose to mate with the males with the least likelihood to produce anti-aphrodisiacs. Similarly, she could lower her chemical signal output to decrease the amount of anti-aphrodisiacs overall.

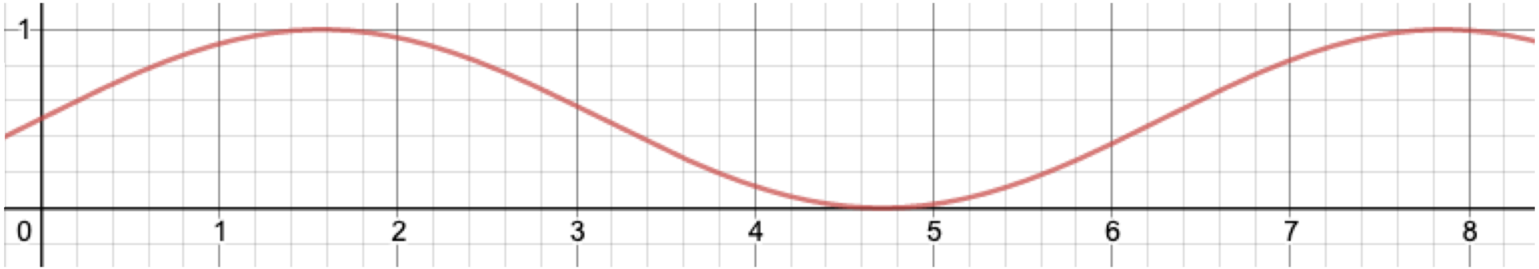
Additional Issues: Part 3

The anti-aphrodisiac depends on the time of day.

We would change our c term, which represents the amount of anti-aphrodisiacs into a function of time rather than a constant.

For example, having the anti aphrodisiac be more effective midday than in the morning or night would make c(t) appear sinusoidal, oscillating through 0 and some peak effectiveness for certain times of day.

$$\frac{\sin(x)}{2} + \frac{1}{2}$$



General Model Improvements

- Consider butterflies' male/female birth ratio to vary beyond 50/50.
  - Would make model more accurate to butterfly birthing behavior.
- Add in an egg population equation.
  - Would model delay in birthing periods more accurately.
- Rearrange the model to remove gender distinction.
  - Would make finding/measuring data to fit to the model easier.
- Add in a separate constant for the butterfly chemical
  - Would allow for more specification between loss of eggs due to female chemicals or male anti-aphrodisiacs

Sources

Cell Press. "A Boy For Every Girl? Not Even Close: Scientists Trace Evolution Of Butterflies Infected With Deadly Bacteria." ScienceDaily. ScienceDaily, 11 September 2009. <www.sciencedaily.com/releases/2009/09/090910121801.htm>

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