Chemical Espionage

Problem C: Chemical Espionage

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Key Players

Pieris brassicae

Trichogramma brassicae

Assumptions

- Exactly ½ the butterfly population is female while the other half is male
- Butterflies go straight from eggs to butterfly
  - Assume there is no death in intermediate larval stages
- Without wasps, butterflies grow without bound
- The only way for the wasp population to grow is through butterfly eggs

Variables and Parameters

B = butterfly population

W = wasp population

a = growth rate of butterflies

k = critical number of wasps

c = probability of butterfly/wasp interaction leading to wasp eggs

m = death rate of wasps

System of Differential Equations

$$\frac{dB}{dt} = aB\left(1 - \frac{w}{k}\right)$$

$$\frac{dW}{dt} = \frac{1}{2}cBW - mW$$

Equilibrium Solutions

- Set derivative equal to 0 and solve
- $\frac{dB}{dt}$: $B = 0 \quad or \quad W = k$
- $\frac{dW}{dt}$: $W = 0 \quad or \quad B = \frac{2m}{c}$

- Extinction Equilibrium: (0,0)
- Coexistence Equilibrium: $(\frac{2m}{c}, k)$

Stability Analysis

$$J(B,W) = \begin{bmatrix} \frac{\partial F}{\partial B} & \frac{\partial F}{\partial W} \\ \frac{\partial G}{\partial B} & \frac{\partial G}{\partial W} \end{bmatrix} = \begin{bmatrix} a - a\left(\frac{W}{k}\right) & -\frac{aB}{k} \\ \frac{1}{2}cW & \frac{1}{2}cB - m \end{bmatrix}$$

Extinction Equilibrium

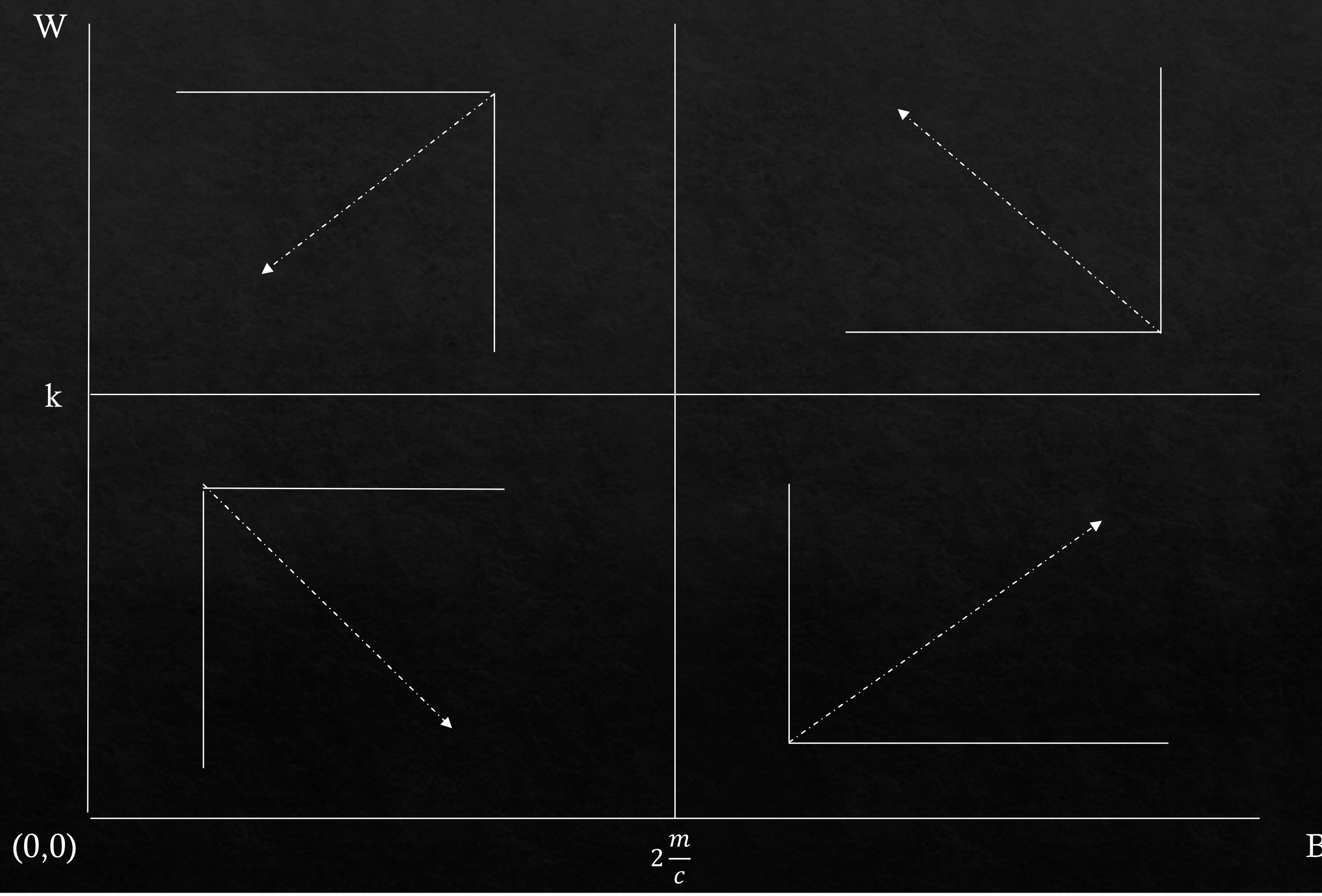
- $J(0,0) = \begin{bmatrix} a & 0 \\ 0 & -m \end{bmatrix}$
- $\lambda_1 = a$
- $\lambda_2 = -m$

- Stable if a<0 but all parameters positive
- Unstable equilibrium

Coexistence Equilibrium

- $J\left(\frac{2m}{c}, k\right) = \begin{bmatrix} 0 & -\frac{2am}{kc} \\ \frac{1}{2}ck & 0 \end{bmatrix}$
- $\lambda_{1,2} = \pm\sqrt{-am}$
- Both equilibrium solutions are unstable
- This is known as the *Principle of Competitive Exclusion*

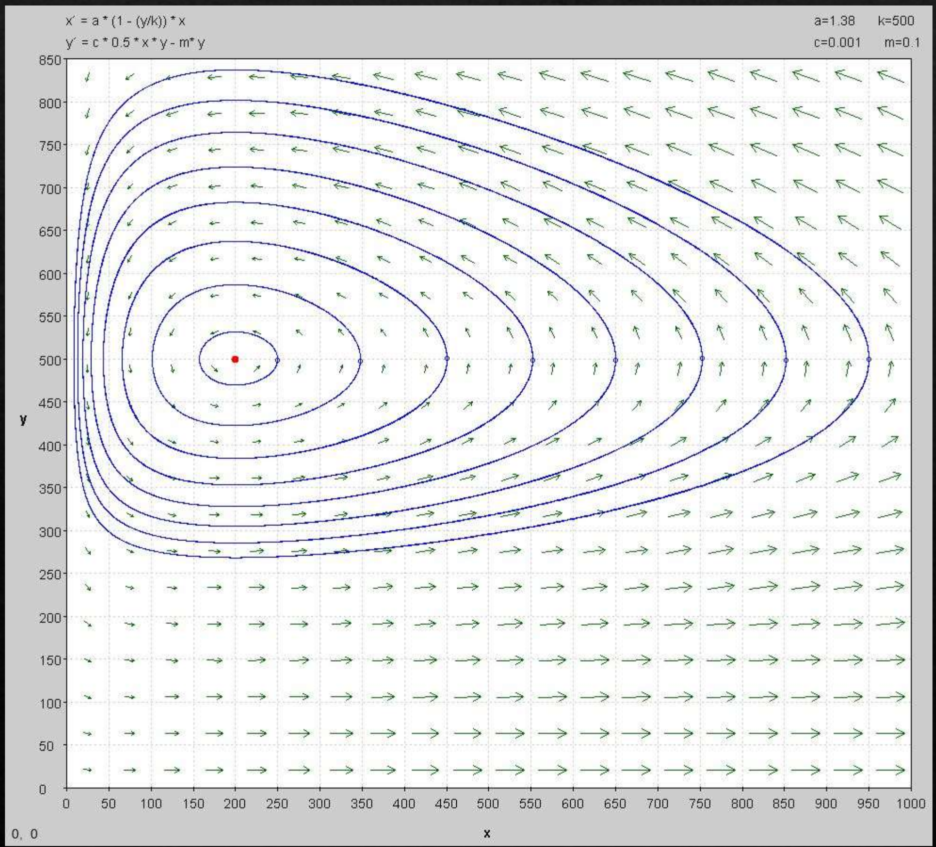
Graphical Analysis



Estimation of Parameters

- $a \approx \frac{Eggs\ laid}{Butterfly\ Lifespan} - \frac{1}{Butterfly\ Lifespan} = \frac{30\ eggs}{21\ days} - \frac{1}{21\ days} \approx 1.38$
- $m \approx \frac{1}{Wasp\ Lifespan} = \frac{1}{10\ days} \approx 0.1$
- k acts as a sort of "carrying capacity" for the butterflies, in terms of wasps. We guessed a reasonable number of wasps for k: 500.
- c was harder to estimate; it represents the interactions between wasps and butterflies, as well as the likelihood that a wasp will mount a butterfly. We estimated c graphically to be approximately 0.001.

Butterfly/Wasp Population Phase Plane



- The coexistence equilibrium is at $(B, W) = (200{,}500)$
- The further the populations get from the equilibrium, the more they fluctuate over time
- The wasp population has a greater effect on the fluctuation of the populations

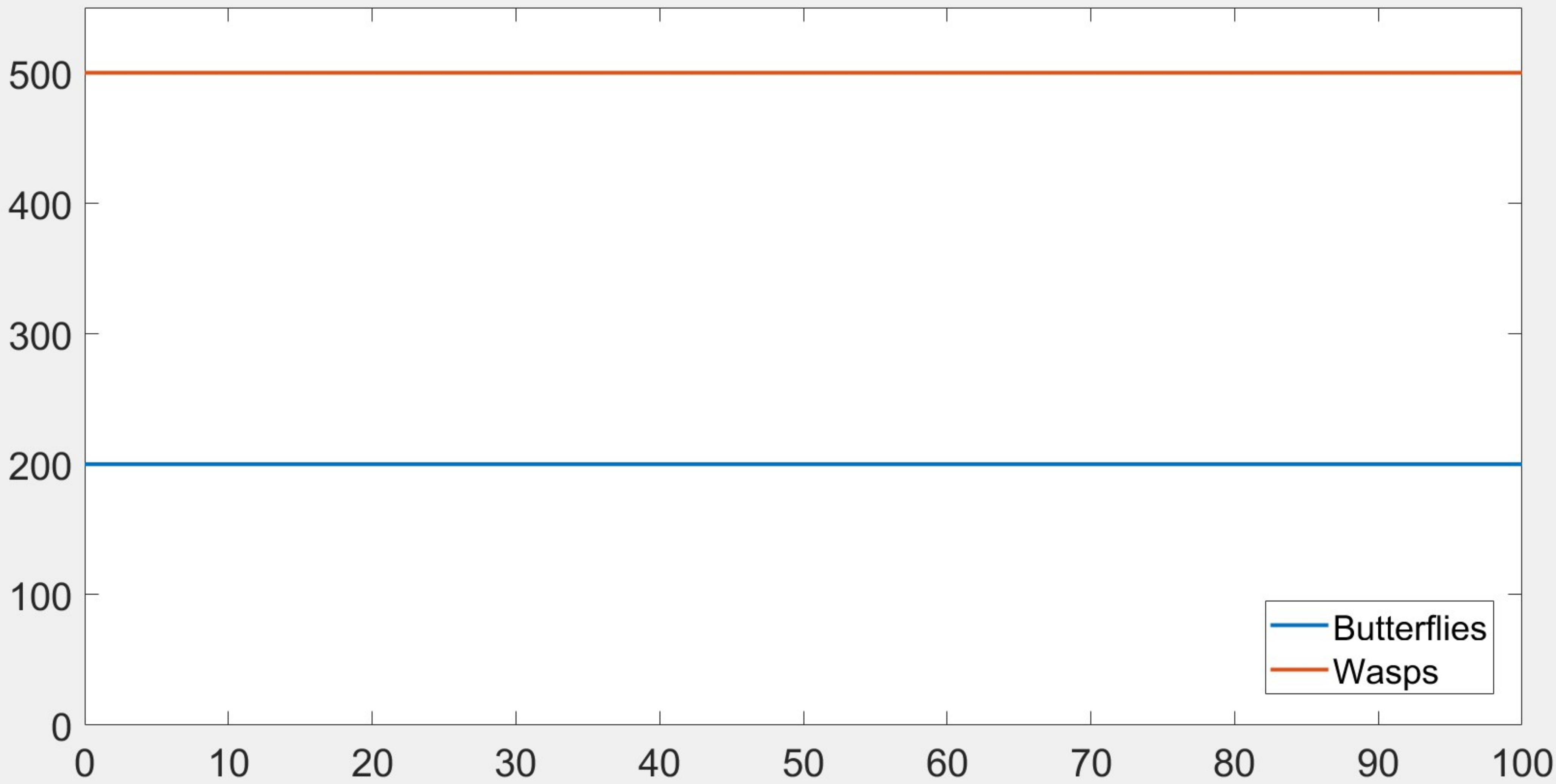
MATLAB Code for Numerical Simulations

comprun.m

```
close all; clear all; clc
global a
global k
global c
global m

a=1.38;
k=500;
c=0.001;
m=0.1;

tspan=[0 100];

x_int=150;
y_int=400;

[T,X]=ode45(@comp,tspan,[x_int y_int]);

plot(T,X(:,1),'linewidth',2)
hold on
plot(T,X(:,2),'linewidth',2)
legend('Butterflies','Wasps')
set(gca,'FontSize',20)
```

comp.m

```
function dxdt = comp(t,x)
  global a
  global k
  global c
  global m

  dxdt(1) = a*(1 - x(2)/k)*x(1);
  dxdt(2) = c*0.5*x(1)*x(2) - m*x(2);
  dxdt=dxdt';

end
```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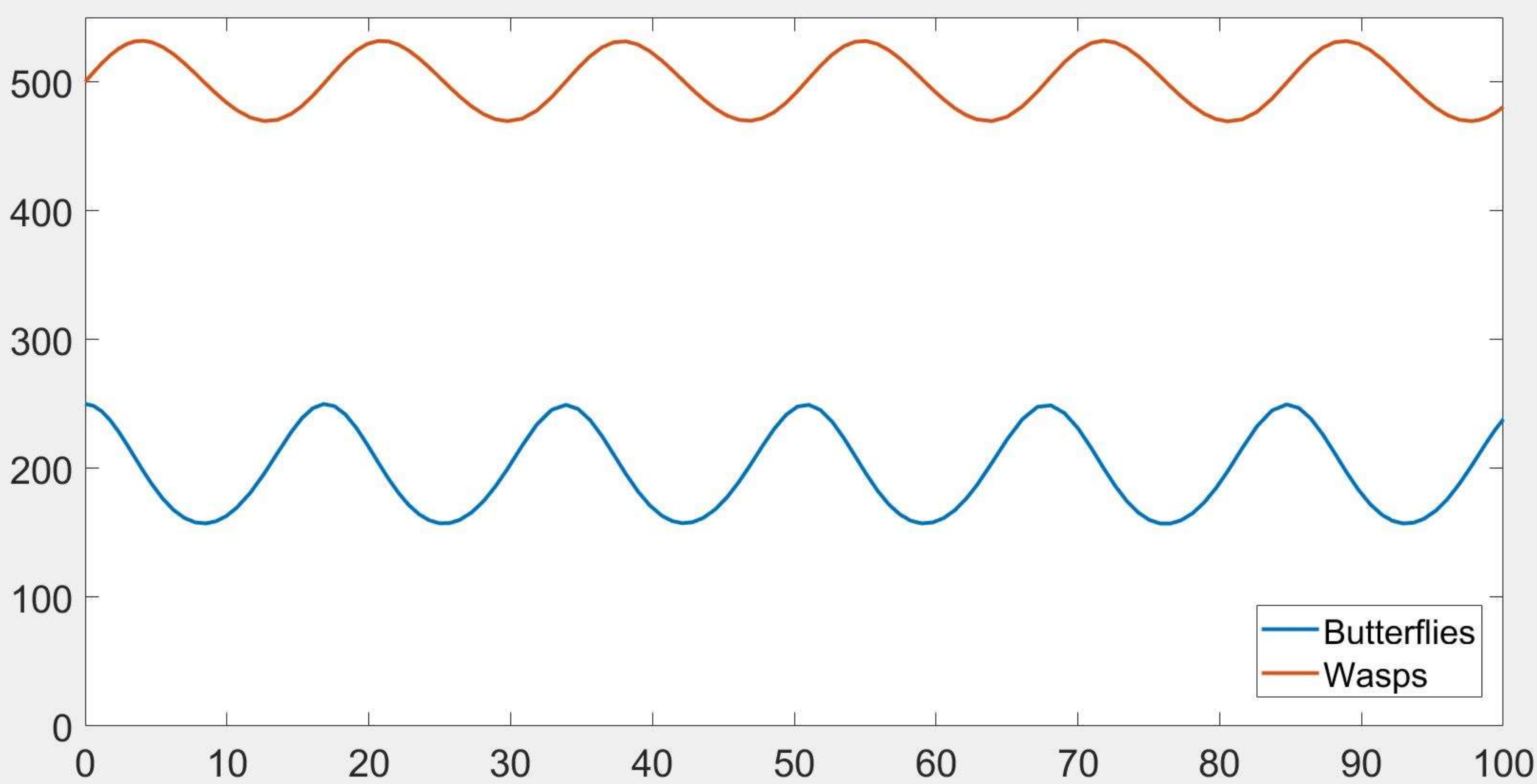
Wasps



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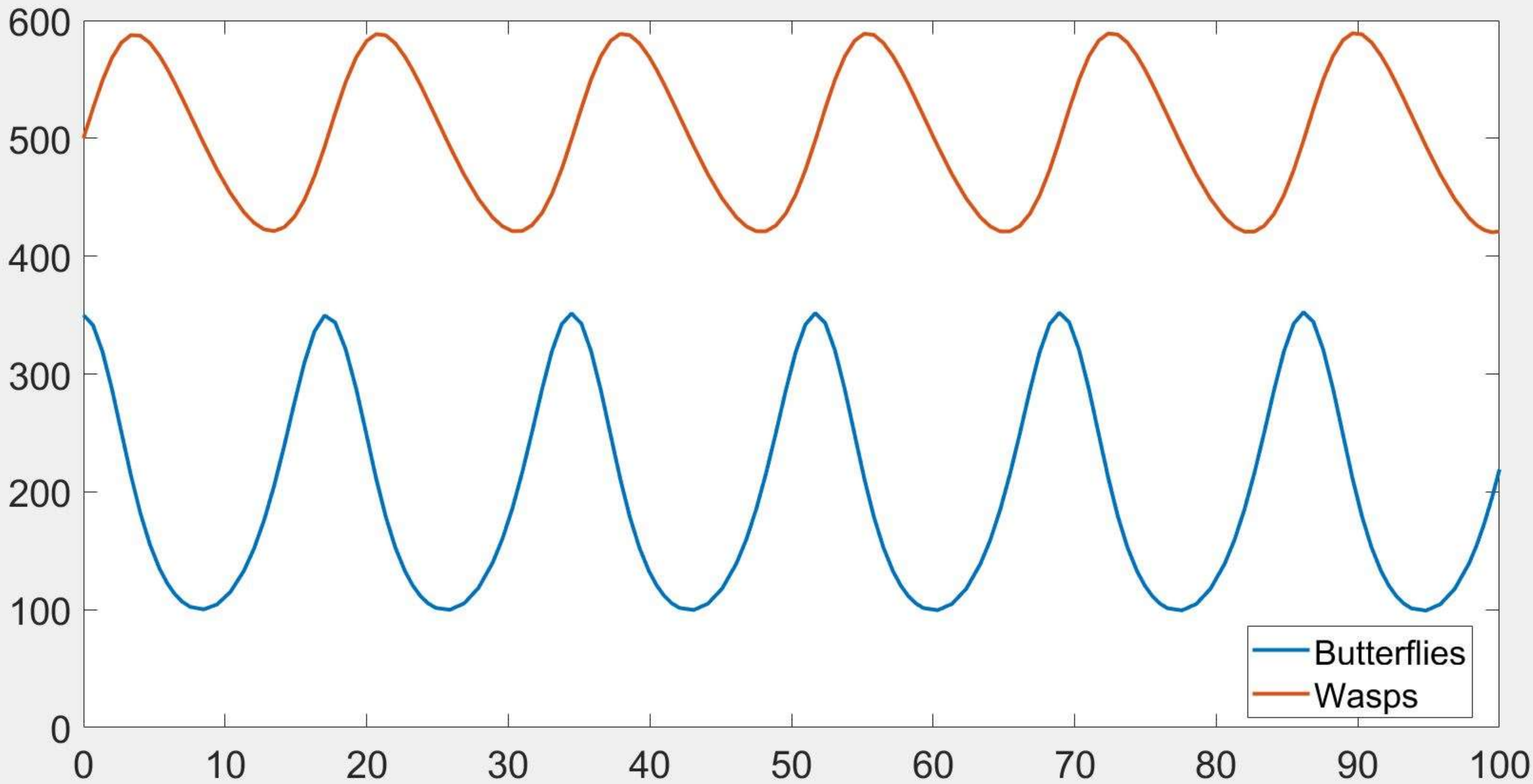
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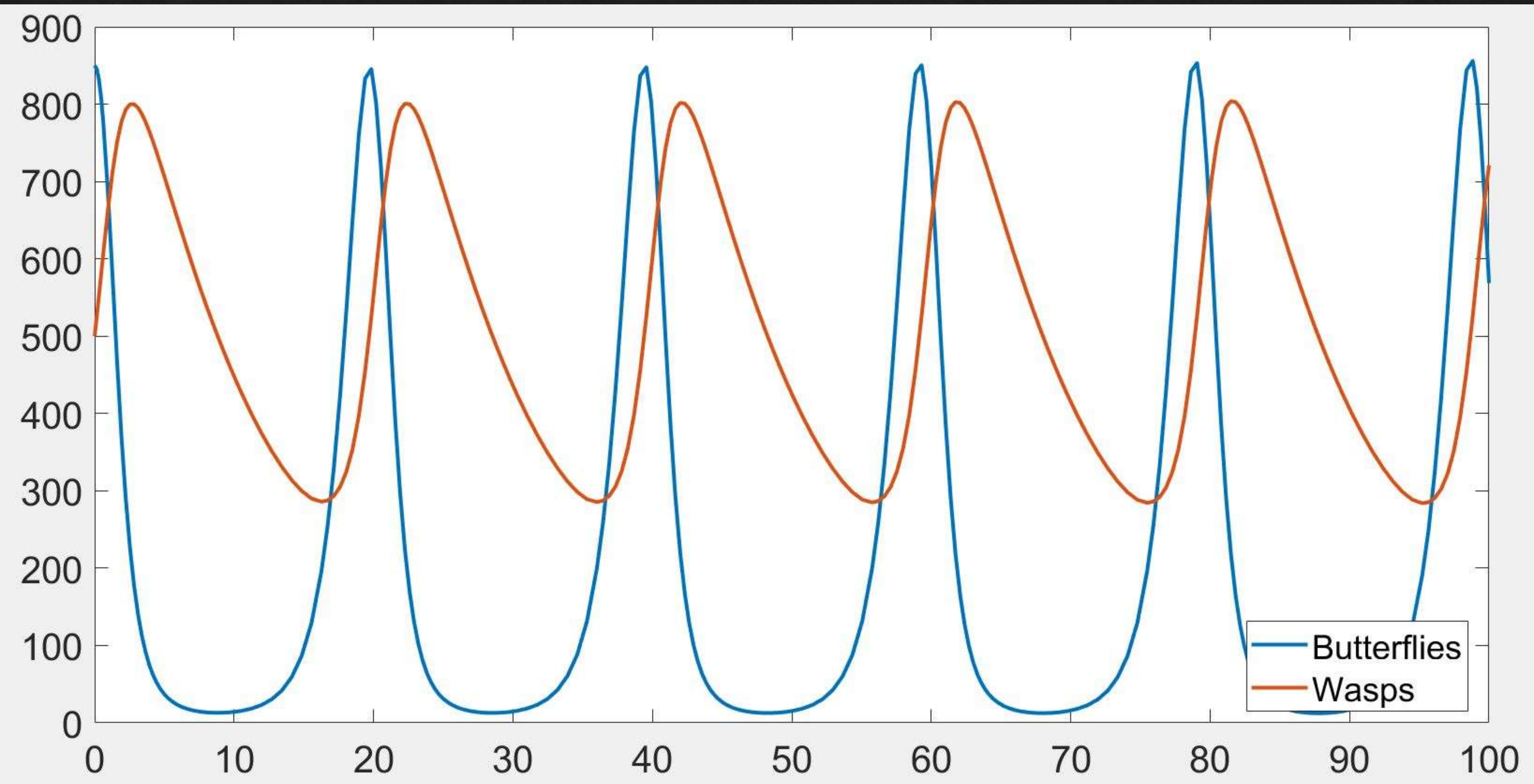

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Butterflies
Wasps

Conclusions

- The parasitic relationship between the wasps and the butterflies is only sustainable if the populations are near the coexistence equilibrium
- A very large initial population of wasps will cause most of the butterflies to die out
- A very small initial population of wasps will cause the butterfly population to explode, which in turn causes the wasp population to explode, which leads to the butterflies dying out very fast
- Either population being too far from the coexistence equilibrium causes large fluctuations in both populations

Additional Issue

- A predator of the both the butterflies and wasps enters the ecosystem

- $\frac{dB}{dt} = aB\left(1 - \frac{w}{k}\right) - \ell PB$
- $\frac{dW}{dt} = \frac{1}{2}cBW - mW - hPW$
- $\frac{dP}{dt} = sPB + rPW - gP$

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- New parameters:
  - ℓ = negative effect of bird predation on butterflies
  - h = negative effect of bird predation on wasps
  - s = benefit to bird for consuming butterfly
  - r = benefit to bird for consuming wasp
  - g = death rate of bird

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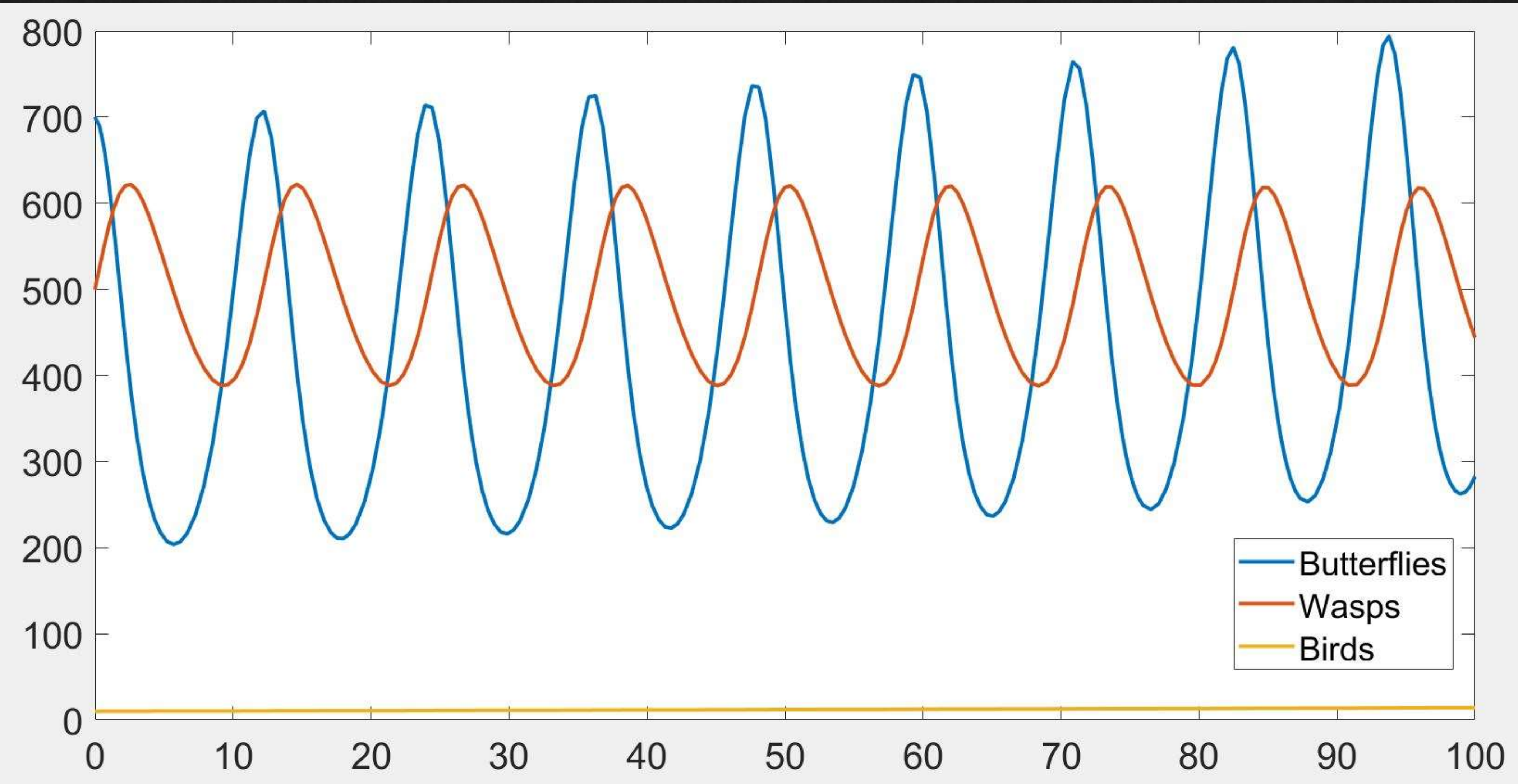
- Notes:
  - Consumption hurts wasps more than butterflies
    - $\ell < h$
  - Consumption affects butterflies and wasps more than birds
  - Small prey, big predator
    - $s < \ell$
    - $r < h$
  - More benefit in consuming butterfly
    - $r \approx \frac{1}{10}s$

Estimation of Parameters

- Negative effect of bird predation on butterfly
  - $\ell = 0.001$
- Negative effect of bird predation on wasp
  - $h = 0.01$
- Benefit of consuming a butterfly
  - $s = 0.00001$
- Benefit of consuming a wasp
  - $r = 0.000001$
- Bird death rate
  - $g \approx \frac{1\ bird}{730\ days} \approx 0.00137$
- Initial conditions were estimated to be close to the coexistence equilibrium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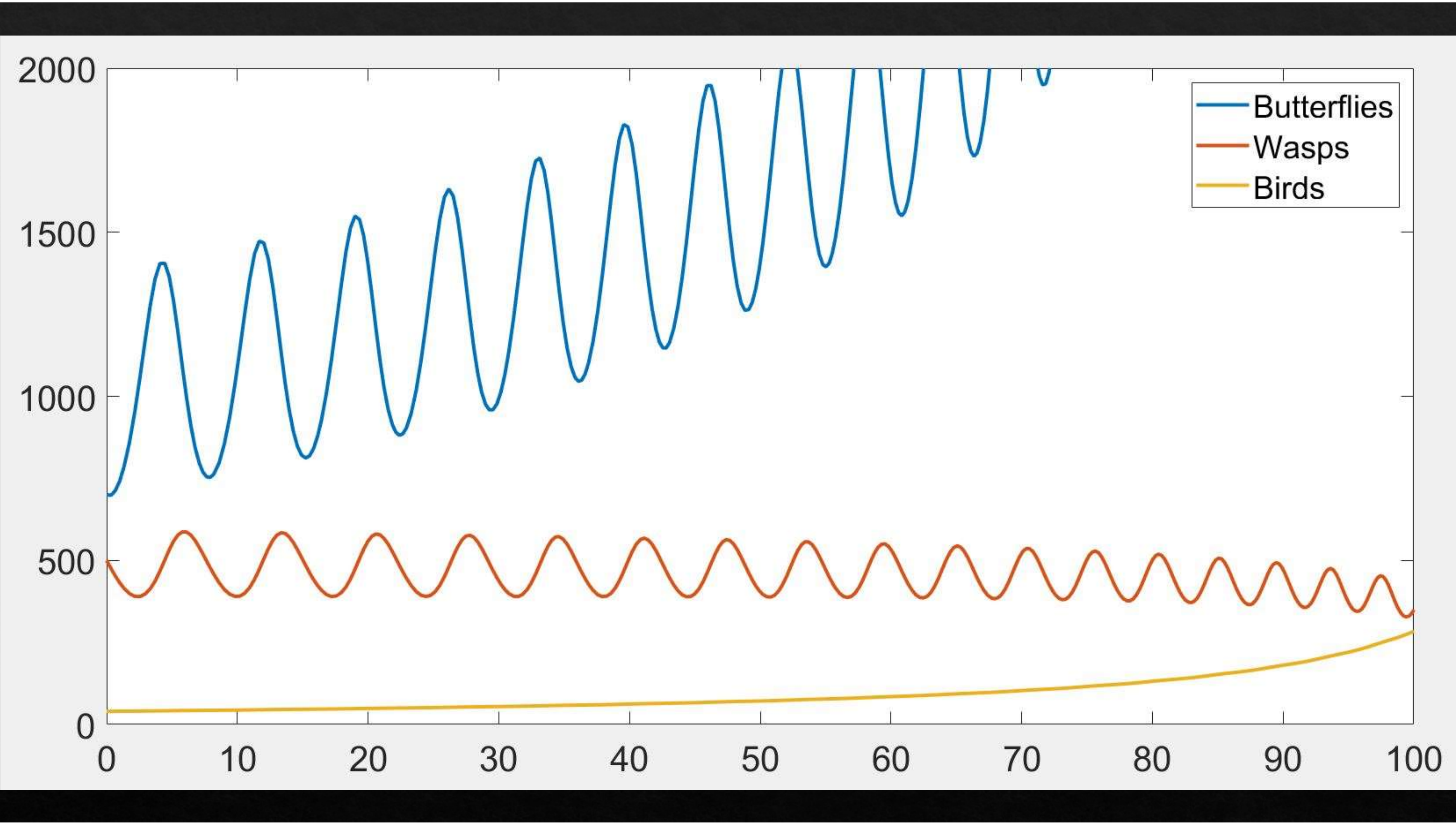
Butterflies
Wasps
Birds



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Butterflies
Wasps
Birds

How our model could be improved

- Account for different larvae stages of the butterfly
- Better way to estimate parameters
- Separate genders of the butterflies and compare their populations