Chemical Espionage (problem c)

DRAKE UNIVERSITY
TEAM 6

The Problem

- Female butterflies use a chemical signals (aphrodisiac) to attract males
- Male butterflies release a different chemical (an anti-aphrodisiac) to deter other males from bothering that female
- Wasps can detect the anti-aphrodisiac and lay eggs in the butterfly eggs so the wasp larva eat the butterfly eggs.
- We want to find a balance in the amount of anti-aphrodisiac produced

Assumptions

- We consider the three populations in isolation, *P.Brassicae* butterflies, *P.Brassicae* eggs, and *Trichogramma* wasps
- Butterflies lay eggs proportional to their population
- Both butterflies and wasps have death rate proportional to their population
- Wasps can only lay their eggs in *P.Brassicae* eggs

Assumptions

- Male butterflies will bother the females during the egg laying process causing the eggs to be suboptimal placed
  - This bothering will be a function of the number of males, the number of eggs and the amount of anti-aphrodisiac produced
- The wasps will hijack the eggs and kill them
  - This hijacking will be a function of the number of eggs and the amount of anti-aphrodisiac
- Eggs that are unbothered and un-hijacked become butterflies at a constant rate

Assumptions

- Butterflies live about twice as long as wasps
- Butterfly eggs are laid in batches of 50
- The ratio of male to female butterflies is approximately constant

Differential Equations

$$\frac{dB}{dt} = \beta E - \delta B$$

$$\frac{dE}{dt} = \alpha B - F(B, E, A) - G(W, E, A) - \beta E$$

$$\frac{dW}{dt} = H(W, E, A) - \gamma W$$

$$F(B, E, A) = c_1 \frac{B^2 E}{1 + A}$$

$$G(W, E, A) = WE(c_2 A + c_4)$$

$$H(W, E, A) = WE(c_3 A + c_5)$$

Equilibrium Solutions

$$0 = \beta E - \delta B$$

$$0 = \alpha B - c_1 \frac{B^2 E}{1 + A} - WE(c_2 A + c_4) - \beta E$$

$$0 = W(E(c_3 A + c_5) - \gamma)$$

$$(0, 0, 0)$$

$$(\sqrt{\frac{(\alpha - \delta)(1 + A)\beta}{c_1 \delta}}, \frac{\delta}{\beta}\sqrt{\frac{(\alpha - \delta)(1 + A)\beta}{c_1 \delta}}, 0)$$

$$(\frac{\gamma\beta}{\delta(c_3 A + c_5)}, \frac{\gamma}{c_3 A + c_5}, \frac{\alpha B - c_1 \frac{B^2 E}{1+A} - \beta E}{E(c_2 A + c_4)})$$

Jacobian Matrix

- We look at the eigen-values of the Jacobian matrix

$$f(B,E,W) = \beta E - \delta B$$

$$g(B,E,W) = \alpha B - c_1 \frac{B^2 E}{1+A} - WE(c_2 A + c_4) - \beta E$$

$$h(B,E,W) = W(E(c_3 A + c_5) - \gamma)$$

$$\begin{bmatrix} \frac{\partial f}{\partial B} & \frac{\partial f}{\partial E} & \frac{\partial f}{\partial W} \\ \frac{\partial g}{\partial B} & \frac{\partial g}{\partial E} & \frac{\partial g}{\partial W} \\ \frac{\partial h}{\partial B} & \frac{\partial h}{\partial E} & \frac{\partial h}{\partial W} \end{bmatrix}$$

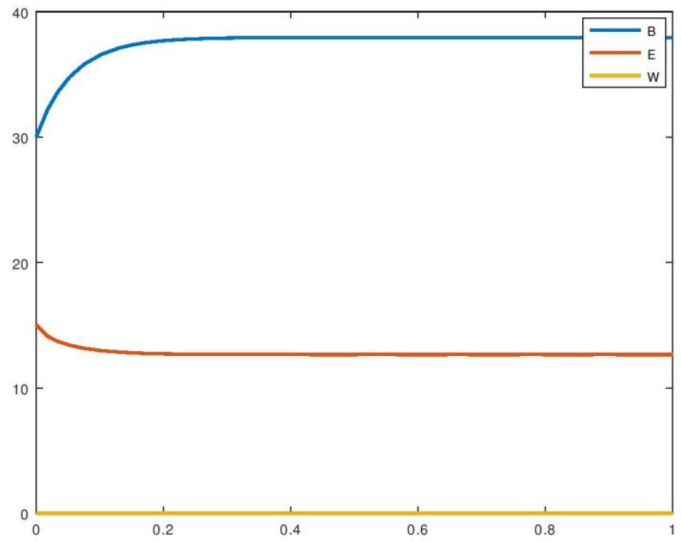
Stability Analysis

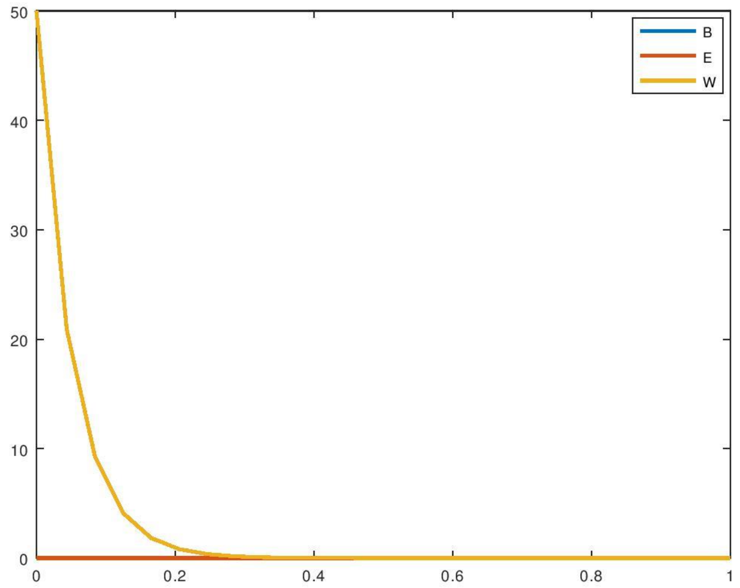
```
bdb=@(B,E,W) -b;
bde=@(B,E,W) a;
bdw=@(B,E,W) 0;

edb=@(B,E,W) c-d*B*E/(1+e);
ede=@(B,E,W) -B*B*d/(1+e)-W*(f*e+g)-a;
edw=@(B,E,W) -E*(f*e+g);

wdb=@(B,E,W) 0;
wde=@(B,E,W) W*(h*e+i);
wdw=@(B,E,W) E*(h*e+i)-j;

x=0;
y=0;
z=0;

A=[bdb(x,y,z) bde(x,y,z) bdw(x,y,z); edb(x,y,z) ede(x,y,z) edw(x,y,z); wdb(x,y,z) wde(x,y,z) wdw(x,y,z)];

eig(A)

x=((c-b)*(1+e)*a/(d*b))^.5;
y=(b/a)*((c-b)*(1+e)*a/(d*b))^.5;
z=0;

A=[bdb(x,y,z) bde(x,y,z) bdw(x,y,z); edb(x,y,z) ede(x,y,z) edw(x,y,z); wdb(x,y,z) wde(x,y,z) wdw(x,y,z)];

eig(A)

x=a*j/(b*(h*e+i));
y=j/(h*e+i);
z=(c*x-d*x*x*y/(1+e)-a*y)/(y*(f*e+g));

A=[bdb(x,y,z) bde(x,y,z) bdw(x,y,z); edb(x,y,z) ede(x,y,z) edw(x,y,z); wdb(x,y,z) wde(x,y,z) wdw(x,y,z)];

eig(A)
```

```
ans =

   22.170
  -72.170
  -20.000

ans =

    -7.9176
  -202.0824
    29.6407

ans =

  -195.6736 +   0.0000i
    -7.1632 +   9.2564i
    -7.1632 -   9.2564i
```

Checks

- We check what happens when the wasp population is 0 and when the butterfly and egg populations are 0





Parameters

α = Egg production rate = 50

β = Egg hatch rate = 30

γ = Wasp death rate = 20

δ = Butterfly death rate = 10

c_1 = proportionality of bothering = 0.1

c_2 = proportionality of wasp hijacking due to anti-aphrodisiac = 16

c_3 = proportionality of wasp hijacking due to random chance = 4

c_4 = proportionality of wasp birth due to anti-aphrodisiac hijacking = 2

c_5 = proportionality of wasp birth due to random hijacking = 0.5

A = amount of anti-aphrodisiac

$$c_2 > c_3$$

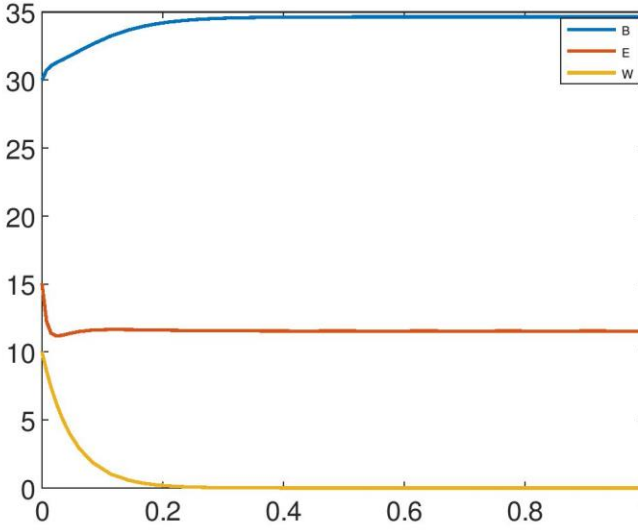
$$\frac{c_2}{c_3} = \frac{c_4}{c_5}$$

$$c_2 >> c_4$$

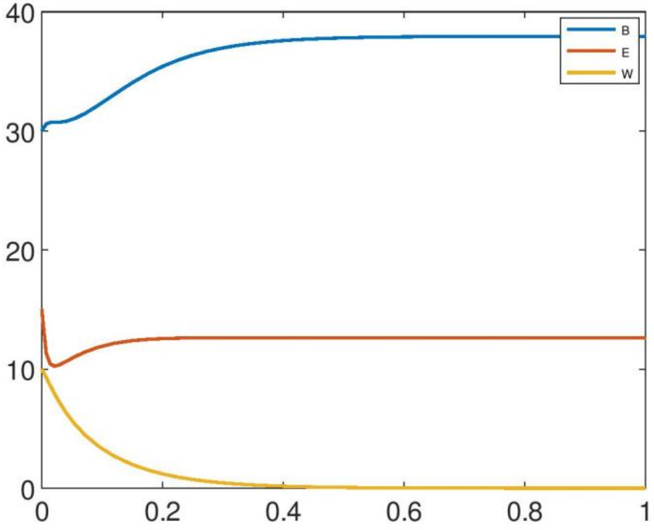
$$c_3 >> c_5$$

Results

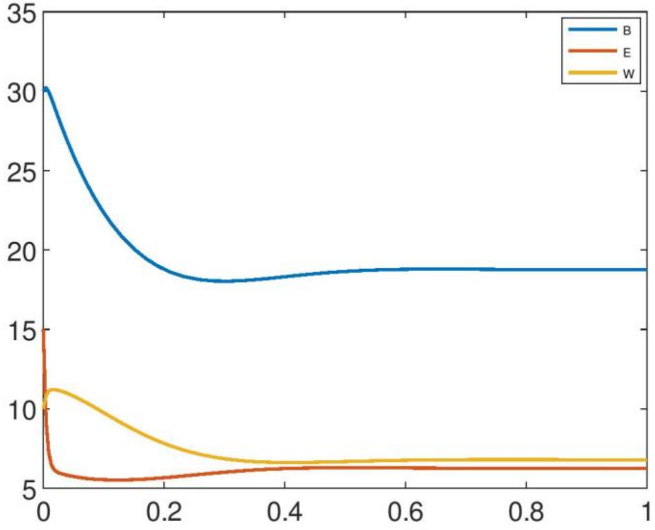
A=0



A=.2

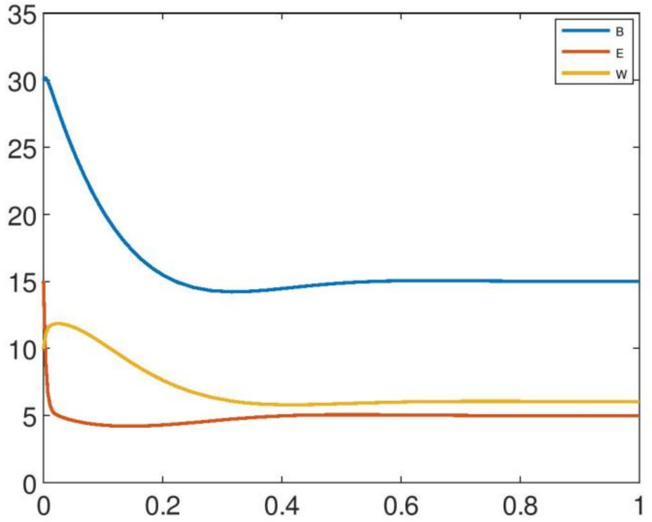


A=.8

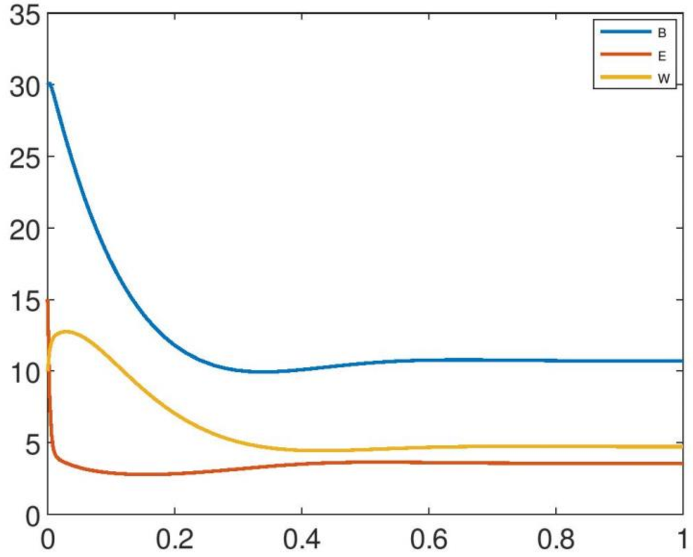


Results

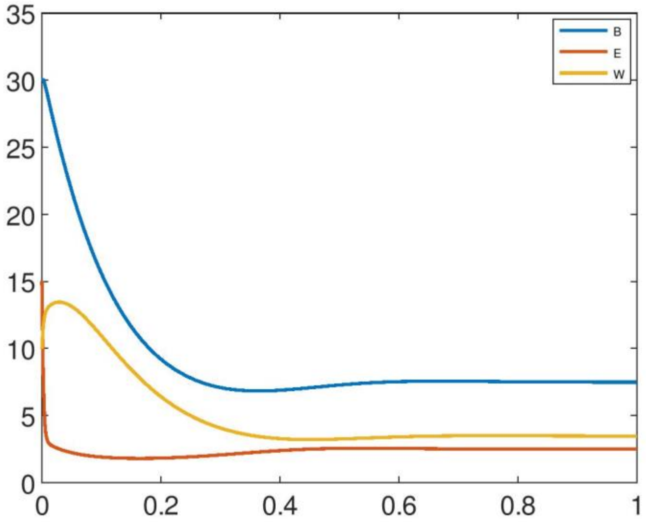
A=1

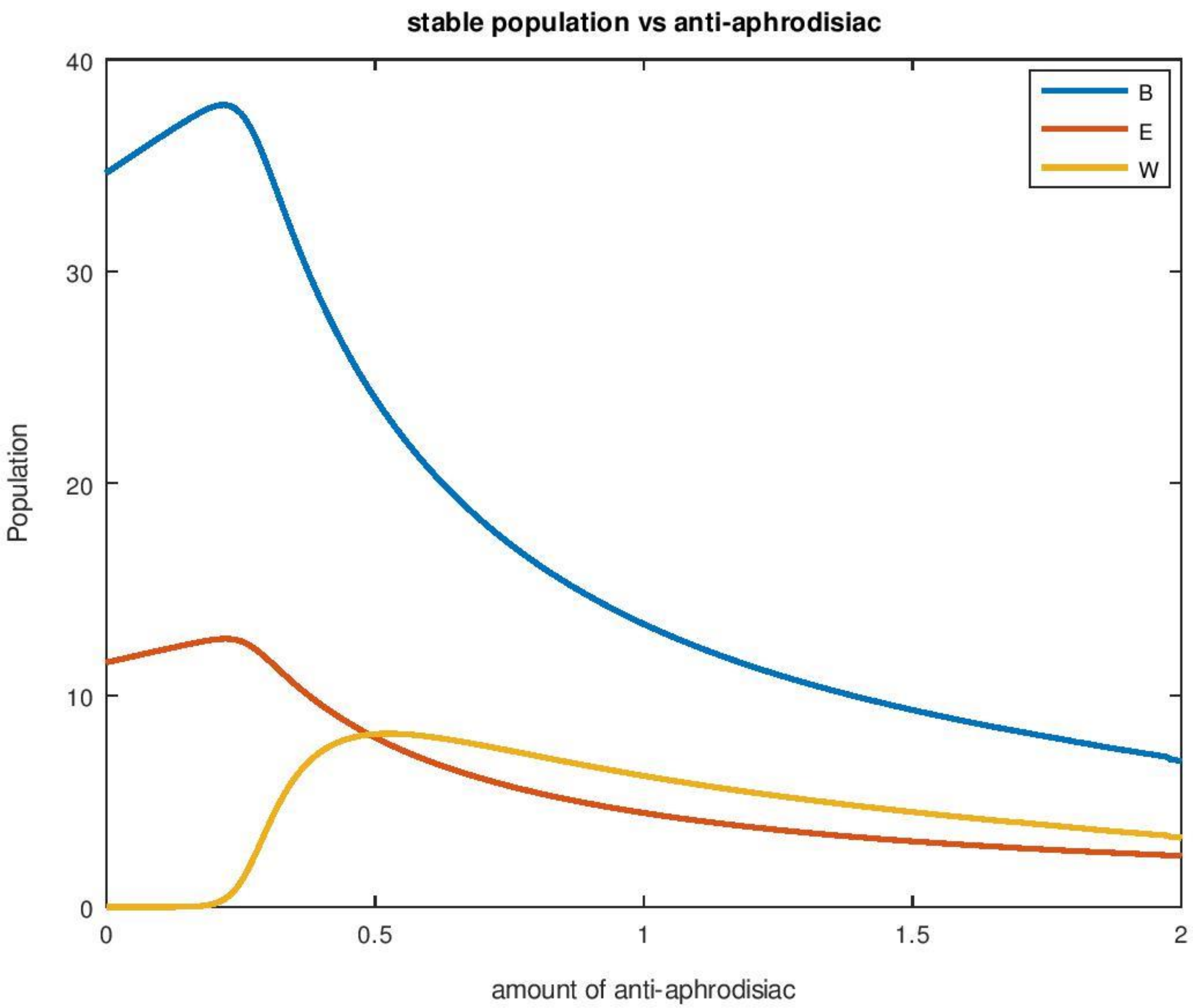


A=1.4



A=2





stable population vs anti-aphrodisiac
Population
amount of anti-aphrodisiac
B
E
W
0
10
20
30
40
0
0.5
1
1.5
2

Improvements

- Seasonal fluctuations of birth rates
- Wasps may be able to lay eggs without the butterfly eggs in different species of butterflies
- If we had population data we could better match our parameters

Additional Issue

- What happens when a predator of both wasps and butterflies is introduced into the system
- We assume the predator's population is dependent on the butterflies and wasps and will die without it

Analysis of Additional Issue

$$\frac{dB}{dt} = \beta E - \delta B - \rho_3 PB$$

$$\frac{dE}{dt} = \alpha B - c_1 \frac{B^2 E}{1 + A} - WE(c_2 A + c_4) - \beta E$$

$$\frac{dW}{dt} = W(E(c_3 A + c_5) - \gamma) - \rho_4 PW$$

$$\frac{dP}{dt} = \rho_1 WP + \rho_2 BP - \kappa P$$

ρ_1 = proportionality of predator growth due to interaction with wasps

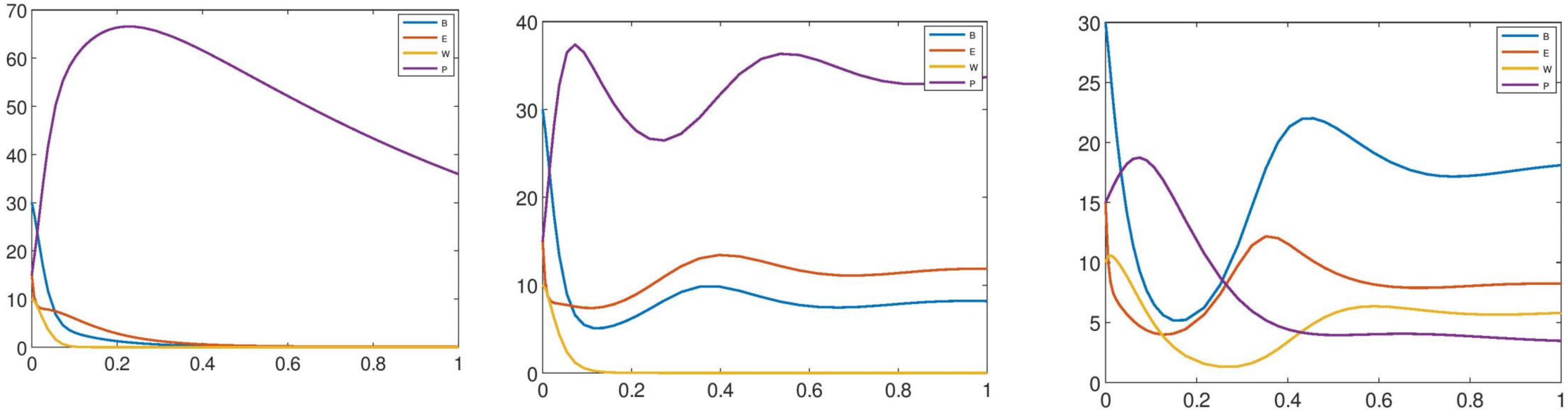
ρ_2 = proportionality of predator growth due to interaction with butterflies

ρ_3 = proportionality of butterfly loss due to interaction with predators

ρ_4 = proportionality of lost due to interaction with predators

κ = Predator death rate

Plots



B
E
W
P
70
60
50
40
30
20
10
0
0.2
0.4
0.6
0.8
1
B
E
W
P
40
30
20
10
0
0.2
0.4
0.6
0.8
1
B
E
W
P
30
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1

References

"Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking *Trichogramma* wasps, "Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros.*Behavioral Ecology*. Volume 21, Issue 3, May-June 2010, Pages 470–478, 11 February 2010. https://doi.org/10.1093/beheco/arq007.

How to Manage Pests. (n.d.). Retrieved November 5, 2019, from http://ipm.ucanr.edu/PMG/NE/trichogramma_spp.html.

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