Problem C

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Florida Southern College
SCUDEM IV 2019

Modelling Population Dynamics of Phoretic Parasitism through Chemical Espionage of Anti-Aphrodisiacs By Parasitic Wasps

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White Cabbage Butterfly and Anti-aphrodisiacs

Huigens, M. E., Woelke, J. B., Pashalidou, F. G., Bukovinszky, T., Smid, H. M., & Fatouros, N. E. (2010). *Behavioral Ecology*, *21*(3), 470-478.

Chemical Espionage

Huigens, M. E., Woelke, J. B., Pashalidou, F. G., Bukovinszky, T., Smid, H. M., & Fatouros, N. E. (2010). *Behavioral Ecology*, *21*(3), 470-478.

Assumptions During Modelling

- Genetic Consistency
- Successful Attack Rates
- Consistent Birth Rates
- No Migration
- Exclusive Reproduction Methods

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Model for Population Dynamics

$$\partial B/\partial t = [(iB - I_w)\, e_i - d_B B]\,(B_{max} - B)$$

$$\partial W/\partial t = I_w\, e_i\, w_e - d_w W$$

$$\partial Iw/\partial t = (\alpha W - d_B\, I_w)(iB - I_w)$$

Symbol	Meaning
B	Number of butterflies
B_{max}	Maximum butterfly population
W	Number of Wasps
d_B, d_W	Death rates of the insects
α	Rate of wasp parasitism

Symbol	Meaning
e_i	Eggs per impregnation
w_e	Number of wasps born per egg parasitized
i	Percentage of butterfly population impregnated
I_w	Number of butterflies that are impregnated and parasitized by a wasp

*

Model for Population Dynamics

$$\partial B/\partial t = [(iB - I_w)\, e_i - d_B B]\,(B_{max} - B)$$

Amount Born | **Amount Dying** | **Logistic Correction**

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Model for Population Dynamics

$$\partial W/\partial t = I_w e_i w_e - d_w W$$

Amount Born — $I_w e_i w_e$

Amount Dying — $d_w W$

Symbol	Meaning
B	Number of butterflies
B_{max}	Maximum butterfly population
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Model for Population Dynamics

$$\partial I_w / \partial t = (\alpha W - d_B I_w)(iB - I_w)$$

Amount Parasitized — Amount Dying — Logistic Correction

Symbol	Meaning
B	Number of butterflies
B_{max}	Maximum butterfly population
W	Number of Wasps
d_B, d_W	Death rates of the insects
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Symbol	Meaning
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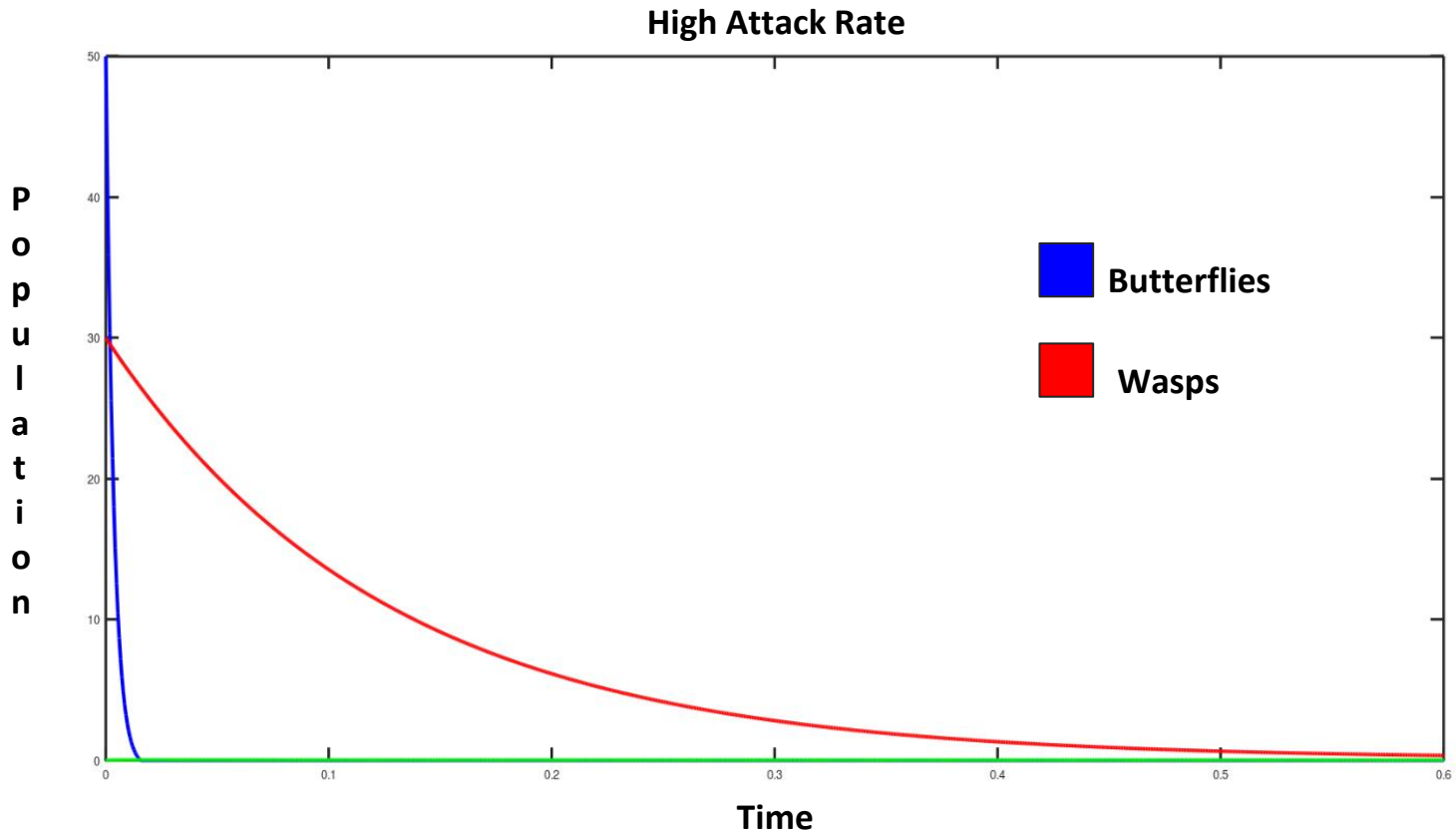
Long Term Behaviors



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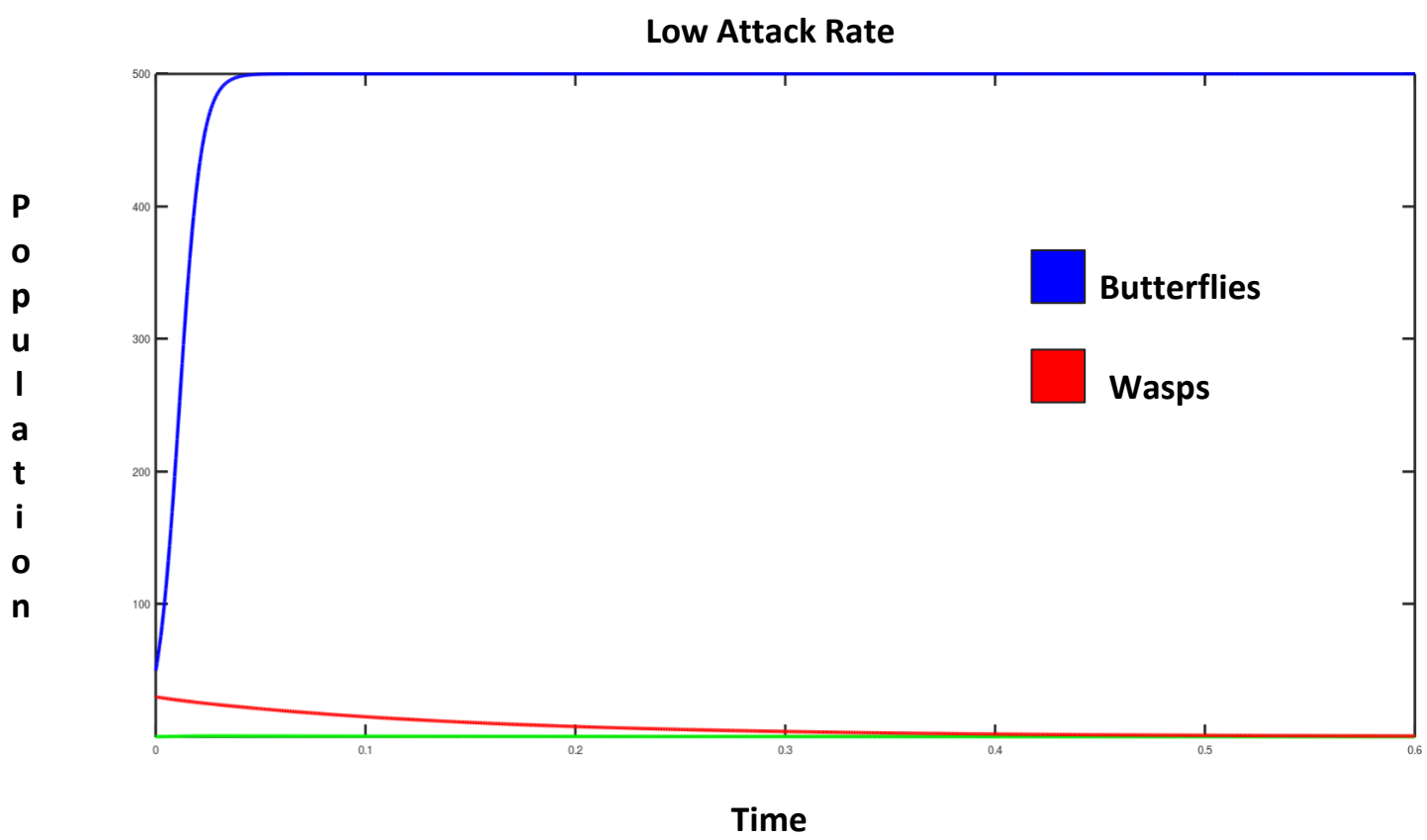
Long Term Behaviors



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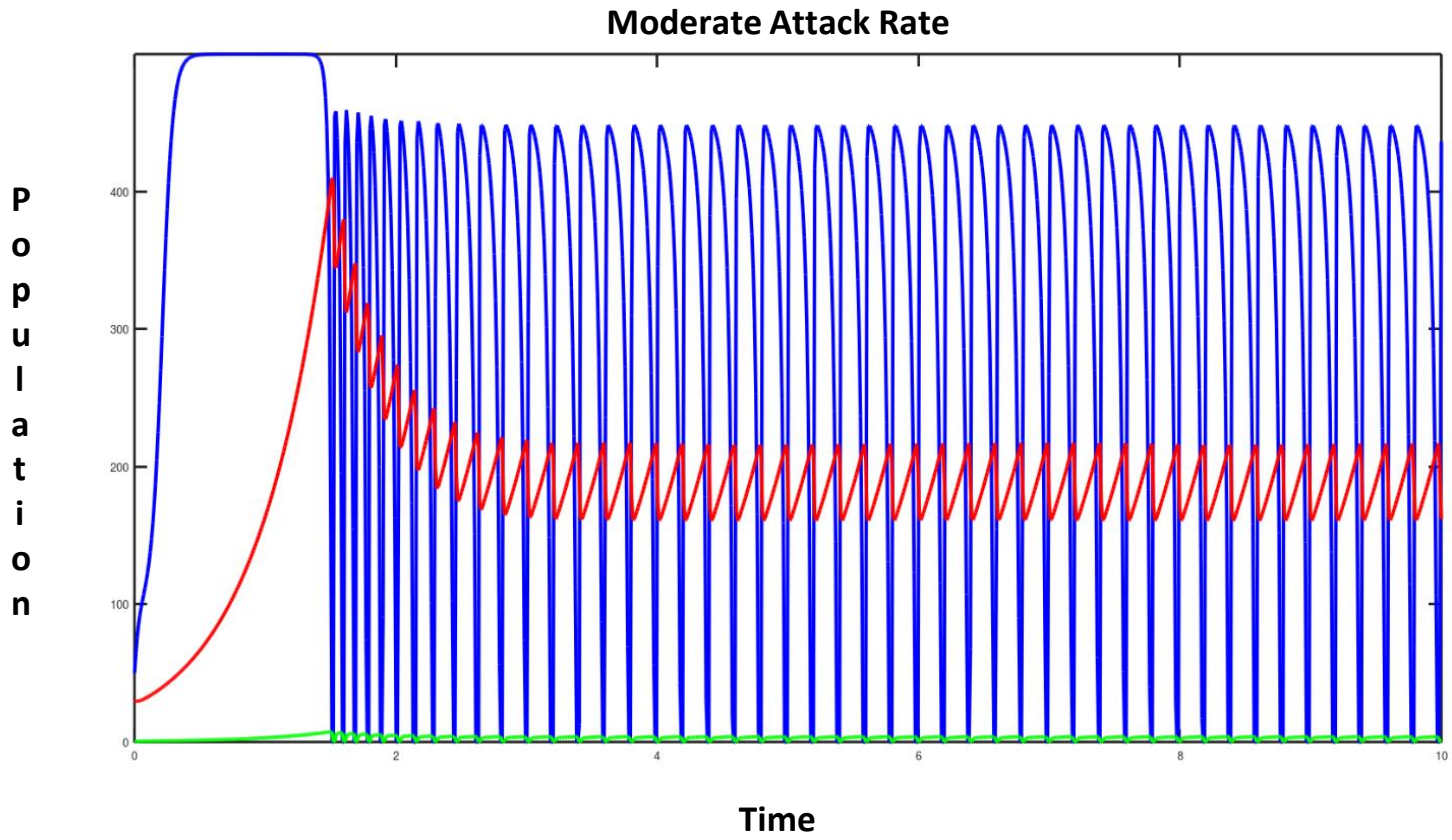
Long Term Behaviors



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Dispersion of Pheromones

$\boldsymbol{\eta}$ = viscosity of air =1.81×10^{-5} kg/(ms)

a = pheromone radius = 1.25×10^{-4} m

V(0) ≅ 0.1 m/s

M = 2$\boldsymbol{\mu}$g

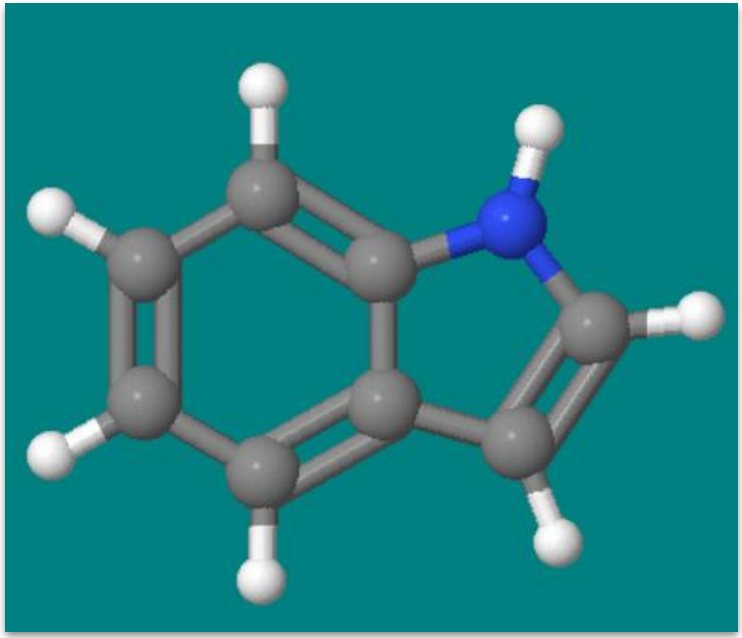
$\boldsymbol{\gamma} = 6\boldsymbol{\pi\eta a} = 4.26 \times 10^{-8}$ kg/s

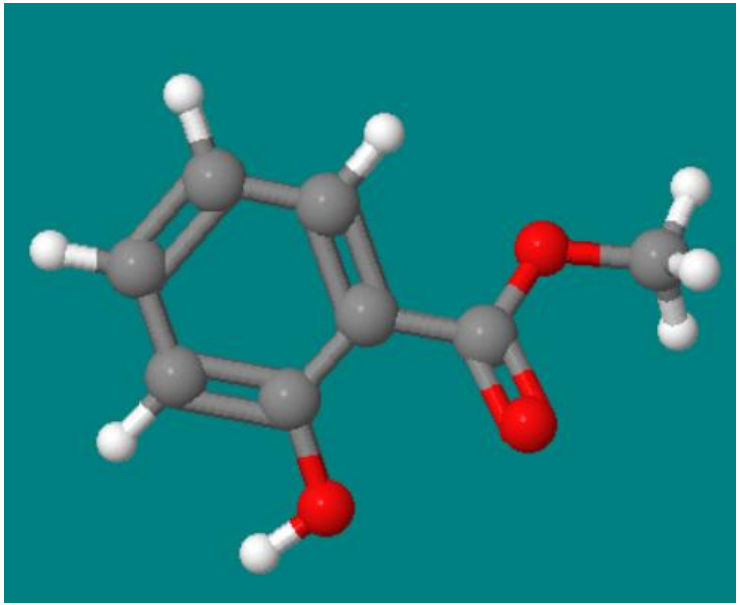
$\mathbf{V(t) = V(0)e^{-tm/\gamma}} = (0.1 \text{ m/s})e^{-t/21.3}$

$\mathbf{X(t)=|V(0)(-\gamma/m)e^{-tm/\gamma}|=|}(0.1 \text{ m/s})(-21.3 \text{ 1/s})\mathbf{e^{-t}}/21.3|$

https://www.engineersedge.com/physics/viscosity_of_air_dynamic_and_kinematic_14483.htm

Indole (L) and Methyl Salicylate (R)





https://chemagic.org/molecules/amini.html

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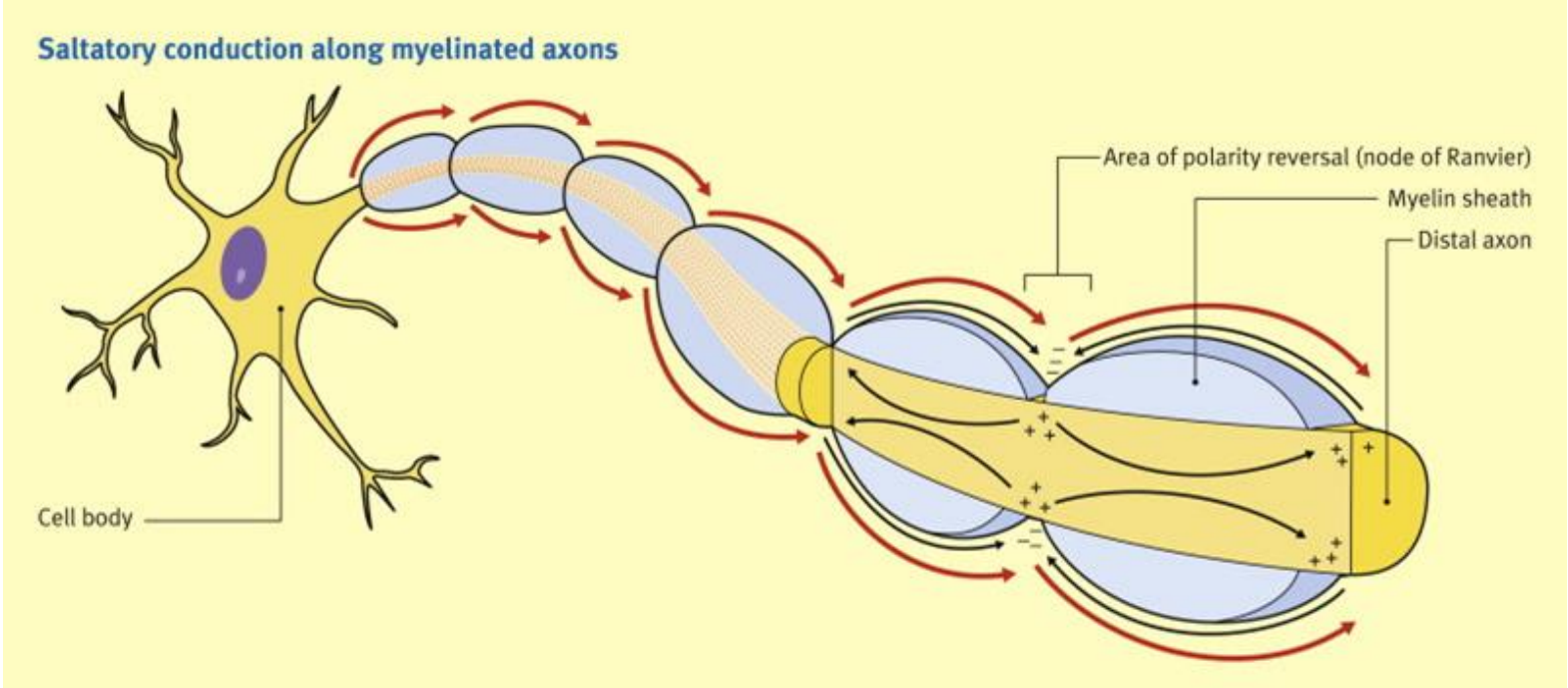
Propagation of the Action Potential



Fletcher, A. (2011). *Anaesthesia & Intensive Care Medicine*, *12*(6), 258-262.

Propagation of the Action Potential

$$\partial V/\partial x = -L\,\partial I/\partial t \text{ and } \partial I/\partial x = -C\,\partial V/\partial t$$

$$\partial^2 V/\partial t^2 - u^2(\partial^2 V/\partial x^2) = \partial^2 I/\partial t^2 - u^2(\partial^2 I/\partial x^2)$$

$$u = 1/\sqrt{LC}$$

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Wasp Trajectory

$$\frac{dvx}{dt} = \frac{Fx}{m} - \frac{c}{m}(v_x^2+v_y^2+v_z^2)^{1/2}v_x$$

$$\frac{dvy}{dt} = \frac{Fy}{m} - g - \frac{c}{m}(v_x^2+v_y^2+v_z^2)^{1/2}v_y$$

$$\frac{dvz}{dt} = \frac{Fz}{m} - \frac{c}{m}(v_x^2+v_y^2+v_z^2)^{1/2}v_z.$$

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Additional Issue

$$\partial B/\partial t = [(iB - I_w)\, e_i - d_B B - d_P B]\,(B_{max} - B)$$

Amount Born — $(iB - I_w)\, e_i$

Amount Dying — $d_B B - d_P B$

Logistic Correction — $(B_{max} - B)$

$$\partial W/\partial t = I_w\, e_i\, w_e - d_w W - d_P W$$

Amount Born — $I_w\, e_i\, w_e$

Amount Dying — $d_w W - d_P W$

Limitations and Future Directions

- Long Term Evaluation of Pheromone Propagation
- Limitations from Assumptions
- Equations of Aerodynamic Flight
- Expansion of Population Dynamics Equations

Thank You for Your Time

Any Questions?