Chemical Espionage Problem C

Cara Cao, Alyssa Hirani, Parul Rai
Coach: Lawrence Udeigwe
Manhattan College

Problem Statement

- Describing the interactions between parasitic wasps, and the butterfly population
- Anti- aphrodisiacs is used by male butterflies to discourage other males from females
- Wasps have developed the ability to exploit this chemical
- Birth rates are in danger due to the wasps
- How will this affect the long term butterfly population

Assumptions

- Assume logistic growth
- Model is based on the concentration of aphrodisiac and no other factors
- Assume closed environment
- Model only describes interactions between the wasps and butterflies
- Assume wasps population is consistent

Growth Rate, R

- R, represents the net effect of the anti-aphrodisiac.
- Assume R is solely dependent on the concentration of anti-aphrodisiac
- Number of eggs consumed > Number of eggs fertilized
- Therefore rate of growth is always negative

$$R = \frac{\#\ of\ eggs\ fertilized - \#\ eggs\ consumed}{total\ \#\ of\ eggs\ fertilized}$$

Derivation of the Formula

Logistic Differential Equation = $\frac{dP}{dt} = KP(1 - \frac{P}{K})$

Analytical solution is obtained by integrating both sides

$$P(t) = \frac{K}{1+Ae^{-Rt}}$$

Where A = $(K-P_0)/P_0$

T = time

P_0 = Initial Population in millions

P(t) = Population at given time t

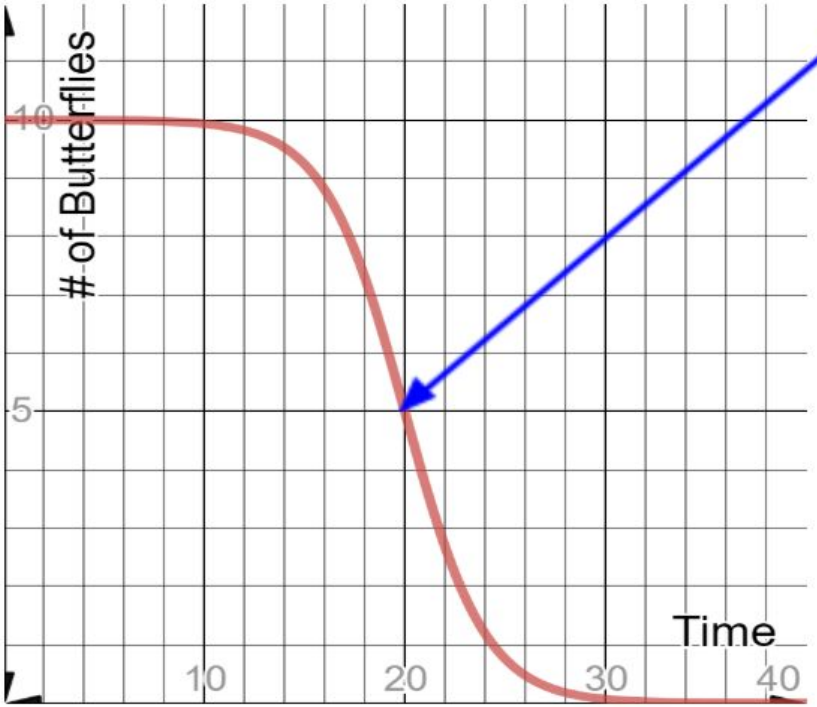
K = Carrying Capacity

Result

- Assumed the current time is the inflection point of the graph
- Shifted the graph horizontally to the right by 20 years
- We assumed:
- K = 10 million butterflies
- P_o = 5 million butterflies
- R = -0.5
- T = x in years

$$P(t) = \frac{10}{1 + (\frac{10-5}{5})e^{-(-0.5)(t-20)}}$$

Graph



$$\frac{d^2P}{dt^2} = 0$$

Conclusion

- The model predicts butterfly population will decay over time
- Inflection point:
  - the rate of female butterflies fertilizing eggs = the rate of wasps eating the eggs
- Not accounting for the idea that species evolve and adapt to combat ecological problems
- In reality, it is in the nature of all species to overcome a predator like wasps

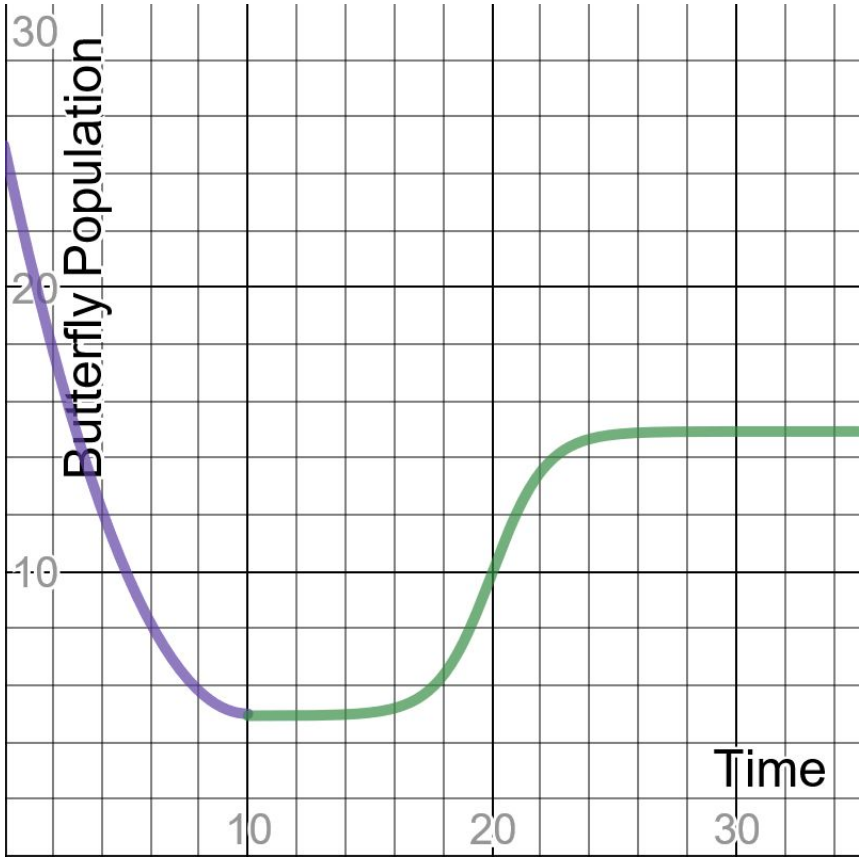
Additional Issues

2) Suppose that the female butterfly could detect a male butterfly's propensity to use the anti-aphrodisiac prior to mating. Based on your model's results what should her strategy be in choosing a mate

- Assume likeliness to release anti-aphrodisiac is a gene
- Assume female butterflies are likely to choose males who are less likely to release anti-aphrodisiac
- R, would change from negative to positive overtime
- Results: the gene for releasing anti-aphrodisiac will die out, the wasps will no longer be a predator, the butterflies will grow exponentially

Graph of the Additional Issue

- Our model becomes a piecewise function
- This leads to the purple line representing exponential decay
- Inflection Point represents the point where the wasps are no longer a predator to the butterflies
- Lowest point represents the point where gene dies out
- Survival of the fittest



Thank you

Questions?