Pieris brassicae and the interconnections of Wasps: Modeled Through Differential Equations

Jacob Siedlecki, Austin Doorlag

November 2019

Abstract

This paper attempts to model butterfly's population relative to wasp attractive anti aphrodisiac. We attempt to model the relationships of these variables through logical deduction. We also demonstrate the effects on these variables in specific common cases. It should be noted that we make the assumption that the population Pieris brassicae is large enough to result in approximately a one to one female to male ratio.

1 Equations and Support

We believe the most appropriate equation to model all relationships within the given system of Pieris brassicae and parasitic wasps would correspond with. It should be noted that we assume the population of Pieris brassicae is large enough to result in nearly a one to one gender ratio.

$$\frac{dM}{dt} = k_1 MF - eW \tag{1}$$

$$\frac{dF}{dt} = k_2 MF - eW \tag{2}$$

$$\frac{dW}{dt} = k_3 MF \tag{3}$$

k_1 is a proportionality constant in (1), $k_1 = 0.01$

k_2 is a proportionality constant in (2), $k_2 = 0.1$

k_3 is a proportionality constant in (3), $k_3 = 0.1$

e is the scaling factor of eggs eaten per wasp. $e = 1$

M is the number of male Pieris brassicae in the ecosystem

F is the number of female Pieris brassicae in the ecosystem

W is the number of wasps attacks that occur

We believe these equations model our system best because they respect all properties a typical ecosystem would normally respect. These equations can be easily tested by plugging in specific values for M, for instance.

In the case $M = 0$

$$\frac{dM}{dt} = 0.01(0)F - eW$$

$$\frac{dM}{dt} = 0$$

$$\frac{dW}{dt} = 0$$

This result is very much expected, as if no males exist the ecosystem will cease to grow. It is also expected that the wasp attacks will cease to occur if there are no male Pieris brassicae because no anti-aphrodisiac is being emitted.

These results can similarly be shown for the female population being zero. ($F = 0$)

2 Lotka–Volterra equations

It should also be noted that our set of equations closely resembles a set of Lotka–Volterra equations. We believe the predator-prey type relationship accurately represents the predatorial relationship of the wasp and Pieris brassicae eggs–resulting in less Pieris brassicae overall.

Our equations reflect this, and can be tested by plugging in a large number for W.

In the instance $M = 100$, $F = 100$, $W = 200$

$$\frac{dM}{dt} = 0.01(100) \cdot (100) - 1 \cdot (200)$$
$$\frac{dF}{dt} = 0.1(100) \cdot (100) - 1 \cdot (200)$$
$$\frac{dW}{dt} = 0.1MF$$

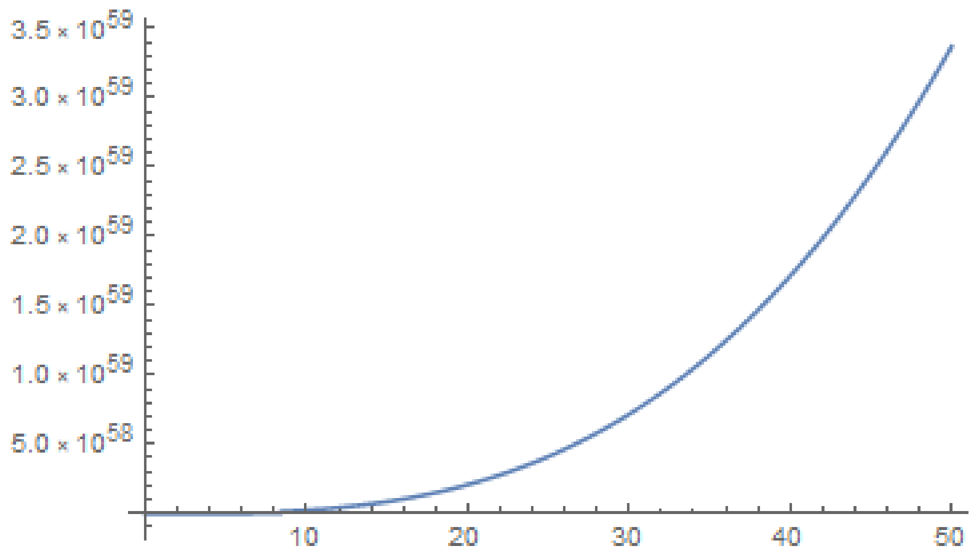
Which results in:

$$\frac{dM}{dt} = -100$$
$$\frac{dF}{dt} = 800$$
$$\frac{dW}{dt} = 1000$$

It is obvious through our model that a large number of wasp attacks will result in a smaller number of male Pieris brassicae being born. It should be noted that the rate at which wasp attacks increases is quite large, and will result in an eventual collapse of the Pieris brassicae ecosystem.

3 Model

We have managed to take our equations and represent them in mathematica. The result of this can be shown below.



This plot reflects our assumption that Pieris brassicae are not dying, and the only variable stunting their growth is wasps eating their eggs. It can easily be displayed as a more typical looking population model if one simply accounts for death in the ecosystem.

4 Interpretation of Model

Based on the model presented above we were able to deduce that the best balance for this system involves a larger population of wasps in order to stunt Pieris brassicae growth. The Pieris brassicae population grows at an extreme exponential rate when left unchecked by a large number of wasps.

In the long run, Pieris brassicae population will reach extreme heights due to lack of wasp interference. This obviously raises the trade-off of Pieris brassicae population versus wasp population. A low wasp population will always result in a larger growth of Pieris brassicae population. With that in mind, it follows that a large wasp population will yield a lower growth of Pieris brassicae.

5 Conclusion

We believe the primary strengths of our model lie in it's inherent simplicity. Our ability to display the relationships of wasps and Pieris brassicae through differential equations allows for us to make conclusions about the properties of this specific relation. Our model does lack some important characteristics however. While many assumptions are made for the sake of simplicity in modeling, we believe this crude model does justice to displaying the interconnections of the ecosystem.