Problem C

Modeling The interactions of P. brassicae and Parasitic Wasps

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Background Information

- The P. brassicae butterfly uses a chemical signal to attract males
  - This can attract too many males so the male butterflies use another kind of chemical signal called anti-aphrodisiacs to prevent this.

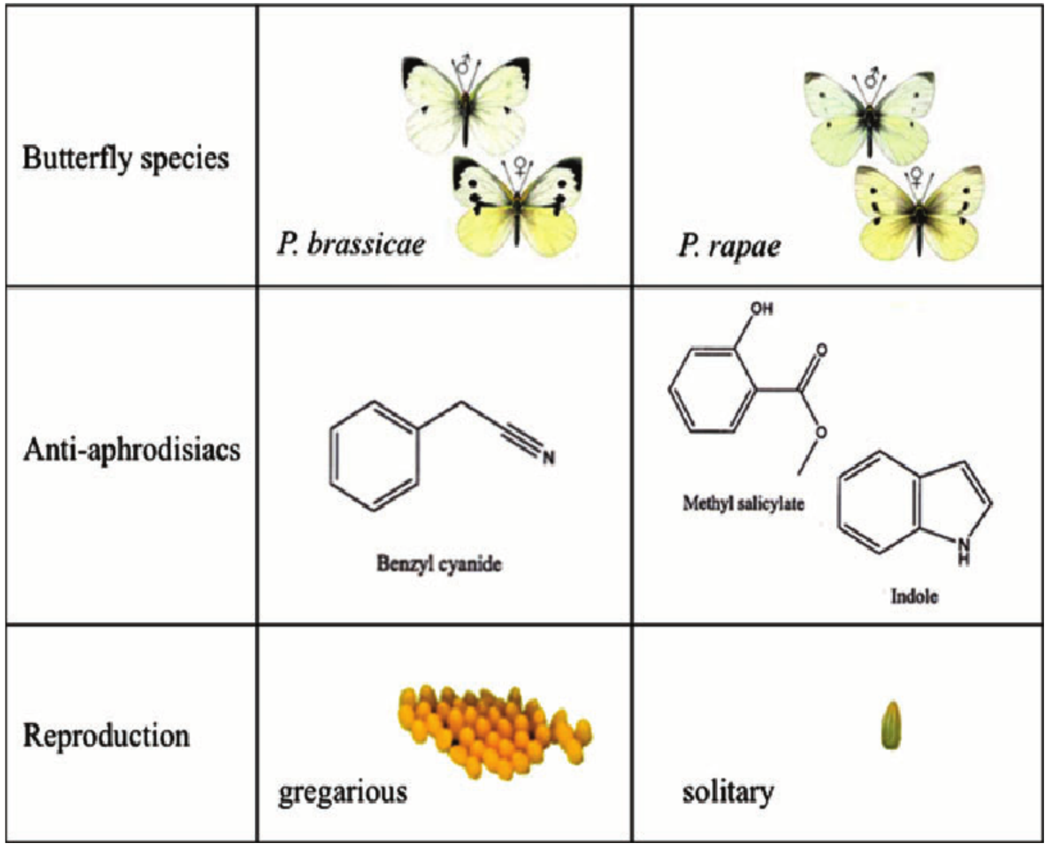
- One problem with this it that these anti-aphrodisiacs can attract parasitic wasps which then lay their eggs in the butterfly eggs.

- We want to model the interaction between the two populations.

Defining Variables

- Let $\mathrm{F}(t)$ be the population of female butterflies at time t
- Let $M(t)$ represent the population of male butterflies at time t
- Left $W(t)$ be the population of wasps at time t
- Let R(t) represent the reproduction between male and female butterflies

Creating $F'(t)$



Putting it all together we get,

$$\boldsymbol{F'(t) = dF(t) + kR(t)}$$

Where $0 \leq k \leq 1$

Creating $F'(t)$

- The current female population is made up of the new butterflies being born and the surviving female butterflies from the last year.

- The current surviving female butterflies are just some fraction of the current $F(t)$
  - $\boldsymbol{dF(t)}$

- These new female butterflies being born are just a proportion of $R(t)$,
  - $\boldsymbol{kR(t)}$

Creating $M'(t)$

- Note that this should be the same as the female population except we need $\boldsymbol{(1-k)R(t)}$ for the male proportion of reproduction

$$\boldsymbol{M'(t) = dM(t) + (1-k)R(t)}$$

Creating $W'(t)$

- We know that the wasp needs to lay its eggs in the butterflies eggs in order to reproduce. This means that the rate at which their population changes depends on the interaction between the eggs (*R'(t)*) and the wasps.

$$\boldsymbol{W'(t) = \theta R(t) W(t)}$$

Where is $\boldsymbol{\theta}$ a constant that can represent the amount of anti-aphrodisiacs used by the female butterflies.

Creating $R'(t)$

- The reproduction is dependent directly upon the male and female interactions which we will show by the following,
  - $\boldsymbol{\alpha F(t)M(t)}$ where alpha is some constant relating to the reproduction

- We also know that the reproduction is partially dependent on how many eggs are killed by the wasps. Knowing this we want to subtract out some proportion of the wasps that will lay eggs.
  - $-\boldsymbol{\delta W(t)}$ where $\boldsymbol{\delta}$ is some proportion of the eggs that are killed or do not make it.

Creating $R'(t)$

- This bring us to the final equation of,

$$\boldsymbol{R'(t) = \alpha M(t)F(t) - \delta W(t)}$$

Where $\boldsymbol{\delta, \alpha \leq 1}$

Note: $\mathbf{W(t)\ and\ R'(t) \geq 0}$

This will come up later…

Analysis of the Equations

- We have created four equations,

$$\boldsymbol{M'(t) = dM(t) + (1 - k)R(t)}$$

$$\boldsymbol{F'(t) = dF(t) + kR(t)}$$

$$\boldsymbol{R'(t) = \ \alpha M(t)F(t) - \delta W(t)}$$

$$\boldsymbol{W'(t) = \ \theta R(t)W(t)}$$

To deal with predator population issue:

Consider,

$$\boldsymbol{R'(t) = \alpha M(t)F(t) - \delta W(t) - a\big(B(t) + W(t)\big)}$$

$$\boldsymbol{B(t) = M(t) + F(t)}$$

Here, ‘a’ is a constant to represent a proportion of butterfly population and wasps population in total.

We assume that the chances of male, female butterflies, and wasps being attacked is equally likely.

What happens when more anti-aphrodisiacs are used?

Real World

- We would expect to see greater mating between males and females (α increases)
- More wasps would be finding and laying eggs (δ increases)
- More wasps are then born (θ increases)

Model

$$\boldsymbol{W'(t) = \theta R(t) W(t)}$$

We know $\boldsymbol{W'(t)}$ will increase

$$\boldsymbol{R'(t) = \alpha M(t) F(t) - \delta W(t)}$$

If,

$$\boldsymbol{\alpha M(t) F(t) = \delta W(t)}$$

Then,

$$\boldsymbol{R'(t) = 0}$$

When, $\boldsymbol{\delta W(t)} >$ $\boldsymbol{\alpha M(t)F(t)}$ then the butterfly population fails to increase but the wasp population increases due to the majority of the eggs being used to birth the wasps.

If this continues, the butterfly population will decrease significantly.

When $M'(t)$ gets small enough then there will be no need for anti-aphrodisiacs

What happens when less anti-aphrodisiacs are used? Here we deal with our second additional issue.

Real World

- We would expect to see slightly less mating between males and females, but this would be insignificant since $M(t)$ is so small.
  - (α stays close to the same)
- The wasps will not be able to locate eggs as easily (δ andθ decrease)

Model

$$\boldsymbol{W'(t) = \theta R(t) W(t)}$$

$\boldsymbol{W'(t)}$ **decreases**

$$\boldsymbol{R'(t) = \alpha M(t) F(t) - \delta W(t)}$$

$\boldsymbol{R'(t)}$ **increases**

End Behavior

The results of the previous slide would cause the population of butterflies to then increase because reproduction is increasing.

Over time as the population of males increases, there will be more of a need to use anti-aphrodisiacs for mating.

This then brings us to the situation where we increase anti-aphrodisiac use and begins a cycle where the two populations are not inversely related, but one will not increase unless the other is increasing.

THANK YOU