Problem C
Chemical Espionage

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Background

Some species of wasps behave as parasites to some species of butterflies. Insects like wasps and butterflies are also a food source for many birds. We wanted to study this host-parasite interaction between *P.rapae* butterflies and *T.Brassicae* as well as the predator-prey relationship between butterflies, wasps and birds to see how the population of the three species affect each other in the long run. Using differential Equations and Euler’s method, we found that the best balance in the system is when bird to butterfly-wasp ratios are very high . We also found that wasps, butterflies and birds die in the long run.

Aim

The questions we were asked to answer are-

- What is the best balance for this system?
- What is likely to happen to both populations in the long run?

Additional Issue-

Adjusting the model to add a predator of both butterflies and wasps

Approach

Initially-

We modeled the system based on

- SIRS models
- Predator-Prey Systems

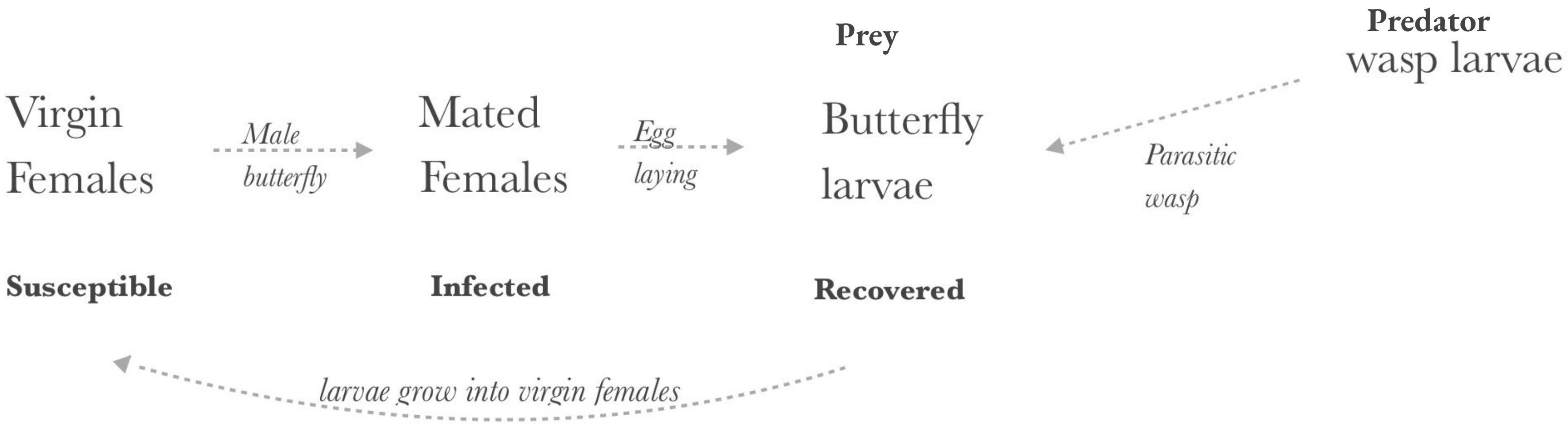
We used Euler’s Method to approximate our solutions.

Assumptions

- We assumed that the system is a 2-species environment with host (butterfly) and parasite (wasp).
- Wasps were modeled to only attack butterfly larvae.
- Mated females to only reproduce once, after which they die.
- Our model also assumes no logistic growth.

Model

SIRS and predator prey (figures)



Model

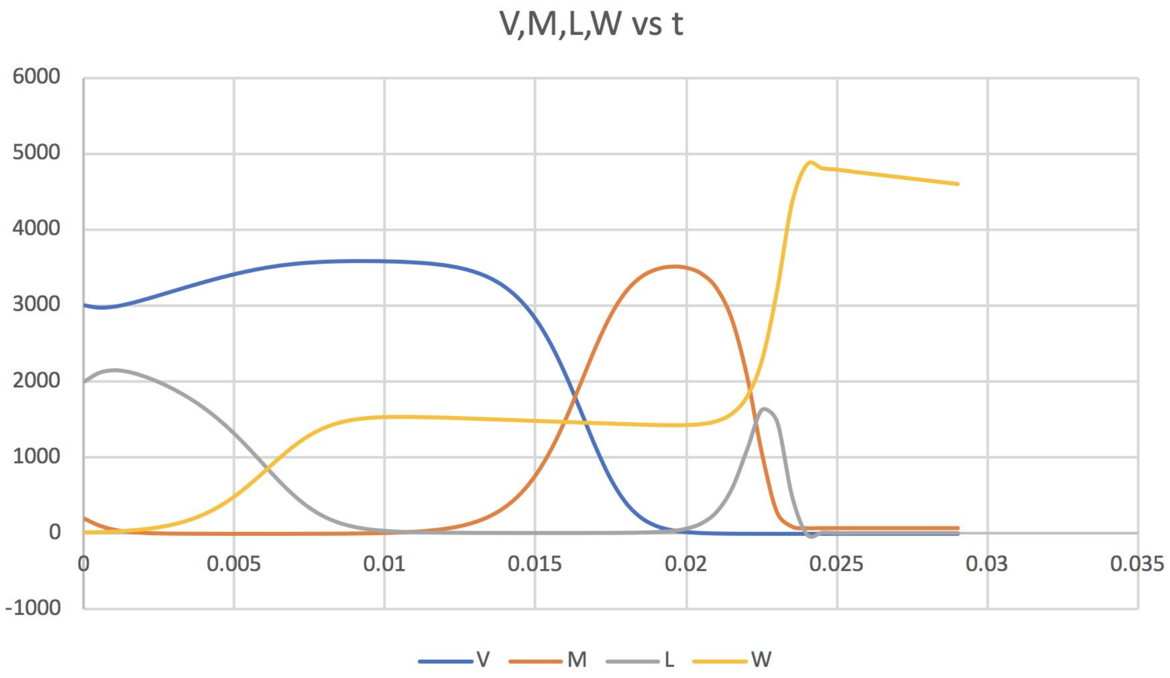
$$dV/dt = -aVM + dLV$$
$$dM/dt = aVM - bLM$$
$$dL/dt = bLM - cLW - dLV$$
$$dW/dt = cWL - eW$$

$V(t)$= number of *female virgin* butterflies at time
$M(t)$= number of *mated female* butterflies at time
$L(t)$= number of butterfly *larvae* at time t
$Wt)$= number of *wasps* at time t

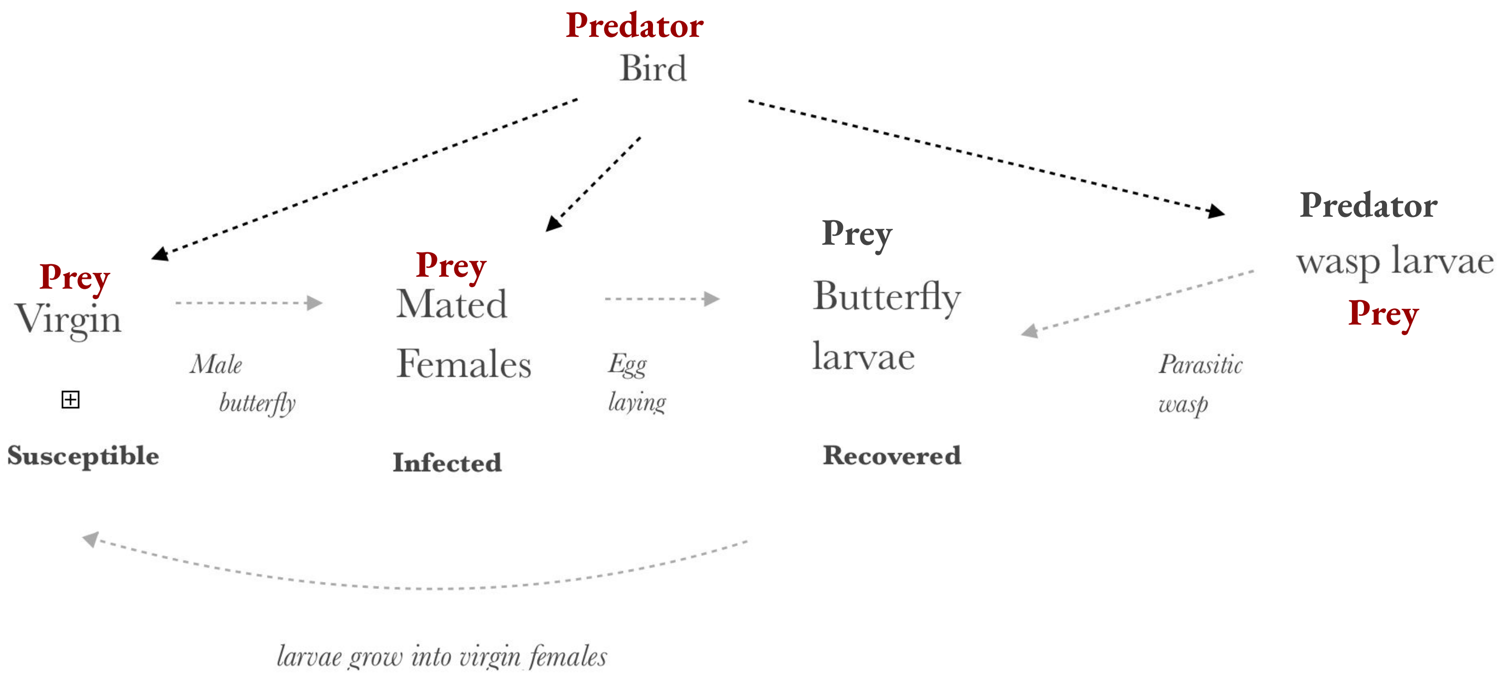
t= time (milli seconds)
a= fertilization rate constant for virgin females
b= death rate constant for mated females
c= susceptibility constant for attack against wasps by mated females
d= growth rate constant of butterfly larvae
e= death rate constant of wasps

Results/Analysis

Graphs (initial)



Model *(adjusted)*



Assumptions (*adjusted*)

- We assumed that the system is a 2-species environment with host (butterfly) and parasite (wasp).
- Wasps were modeled to only attack butterfly larvae.
- Mated females to only reproduce once, after which they die.
- Our model also assumes no logistic growth.
- birds do not discriminate between virgin and mated butterflies
- population will grow naturally (birth and death exist).

Model

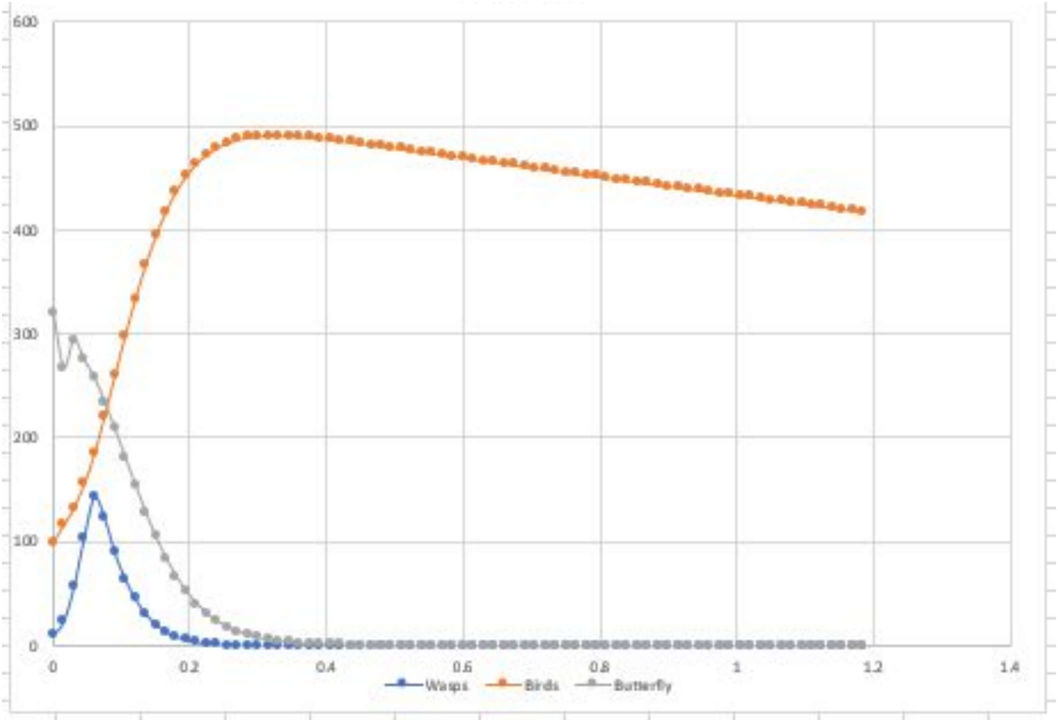
(adjusted)

$$dV/dt = -aVM + dLV - xBV$$
$$dM/dt = aVM - bLM - xBM$$
$$dL/dt = bLM - cLW - dLV$$
$$dW/dt = cWL - eW - yBW$$
$$dB/dt = xBV + xBM + yBW - zB$$

t= time (milli seconds)
a= fertilization rate constant for virgin females
b= death rate constant for mated females
c= susceptibility constant for attack against wasps by mated females
d= growth rate constant of butterfly larvae
e= natural death rate constant of wasps
x=rate at which bird eats butterflies
y= rate at which bird eats wasp
z= natural death rate of birds

Results/Analysis

Graphs (*adjusted*)



Conclusion

As the butterfly pop decreases, wasp pop increases until a certain time *t* after which the wasp pop decreases because they cannot survive without butterfly larvae since they are parasitic.

The bird population rises with an increase in numbers of wasps and butterflies and decreases otherwise.

Once the wasp and bird population starts to decrease, the bird population reaches a maximum and decreases afterwards.

Long term behavior: environment cannot support all species in the long-run (since model doesn’t consider carrying capacity)

Thank you!