SCUDEM IV 2019

PROBLEM CHOSEN: PROBLEM C (CHEMICAL ESPIONAGE)

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CHEMICAL ESPIONAGE

AIM: To use mathematical modelling to determine the best balance for the competing pressure between butterflies and parasitic wasps.

A wasp is an insect that is neither bee nor ant and they reproduce by laying eggs. At the larva stage of their development, they are known as Wasp Larva. Wasp Larva are parasitic in nature, residing either on the body or inside the body of the hosts. It is clear that the Wasp Larva survives by their host. In this case, the hosts are the butterflies' eggs. The wasps larva feed on the eggs to survive the development stage. An equation must be built such that neither party goes into extinction. An equation can be formulated which shows the interaction between male and female butterflies, Wasp Larva and butterfly eggs.

RELATIONSHIP BETWEEN THE BUTTERFLY EGGS, FEMALE BUTTERFLY AND MALE BUTTERFLY OF THE ECOSYSTEM

In this model, we assume the wasp larva are ectoparasitic.

Let $W_l(t)$ denote the number of wasp larva at a time t.

Let b(t) denote the number of butterfly eggs produced at a time t.

Let A denote anti-aphrodisiac

Let F(t) denote the number of female butterflies at a time t

Let M(t) denote the number of male butterflies at a time t

$db/dt = AF$ —— eqn(1)

$dA/dt = kM$ where k is a constant denoting climatic conditions that may affect the antiphrodisiac ——- eqn(2)

From eqn(1), $A = 1/F(db/dt)$ —— eqn(3)

From eqn(3), $dA/dt = 1/F(d^2b/dt^2) - (1/F^2(db/dt))$ —— eqn(4)

combining eqn(2) and eqn(4), we have;

$1/F(d^2b/dt^2) - (1/F^2(db/dt)) = kM$

But the female butterfly is directly proportional to the egg i.e. F α b which implies that F=rb where r is a constant.

Therefore, substituting for F in eqn (5) gives

$\frac{d^2b}{dt^2} - \frac{r}{b}\frac{db}{dt} - Mk\frac{b}{r}$ where k is a constant

Therefore, $\frac{d^2b}{dt^2} - \frac{r}{b}\frac{db}{dt} - Mk\frac{b}{r} = 0$ is our differential equation denoting the interaction between the number of butterfly eggs and female butterfly with initial conditions $b(0) = M_0$ and $b'(0) = b_1$

Solving for the differential equation and assuming that $M \longrightarrow 0$, we obtain the solution $b = b_0 + Db_1 + \frac{1}{2}\frac{Mkb_0 + r^2Db_1}{b_0}r + ...,$

where b_0 denotes the initial population of butterfly eggs.

and b_1 denotes the absence of hazardous conditions.

The solution shows the dependence of butterflies' growth on other conditions.

RELATIONSHIP BETWEEN THE BUTTERFLY EGGS AND THE WASPS OF THE ECOSYSTEM

let W(t) denote the number of wasp at a time t

let $W_L(t)$ denote the number of wasp Larva at a time t

$$\frac{db}{dt} = -cW_L \qquad (*)$$

where c is a constant denoting no adverse environmental influence capable of destroying both the butterfly eggs and wasp larva

$$\frac{dW}{dt} = bW_L \qquad (**)$$

where b denotes the butterfly eggs

Making W_L the subject of the formula in eqn(*) and substituting it into eqn(**) gives

bdb = -cdW with initial condition $b(o) = b_0$

The solution of the above equation gives $b^2 = -2cW + b_0$ where P is a constant

This explains the relationship between the number of butterfly eggs and wasp larva.

The solution also suggests that in the butterflies' relationship with the wasp, the butterfly can only be fully eradicated (i.e. b = 0) only if $b_0 = 2cW$ and W can only be equal to zero (W = 0) only if $b^2 = b_0$

CONCLUSION

Therefore, to create a balance within the system in the long run, the square of the increase in the population of the butterflies' eggs must be twice as much as increase in the population of the wasp larva. This means that the growth of wasps must be curtailed to some degree and more male butterflies be introduced into the system.

ADDITIONAL ISSUES

1. Suppose a bird is added to the system. The bird feeds on both the wasps and the butterflies. From the model, second solution to be precise, then $b^2 = -2cW$

2. The butterfly must choose a male that would not use the antiphrodisiac or use it in luttle proportion to reduce the chances of the wasps tracing where she lays her egg hence, laying their eggs with the butterflies' eggs. From the model, it is advisable she chooses a mate with little or no use of antiphrodisiac.

3. From the first diffeerntial equation, $\frac{d^2b}{dt^2} - \frac{r}{b}\frac{db}{dt} - Mk\frac{b}{r} = 0$ where k signifies the climatic conditions affecting the anti-aphrodisiac. For very large k, it becomes more favorable for the butterfly population. i.e. increase in k favours the butterfly population.

But for small k, i.e. as $k \longrightarrow 0$, the equation becomes $\frac{d^2b}{dt^2} - \frac{r}{b}\frac{db}{dt} = 0$ the solution gotten from this differential equation is the balance needed in the ecosystem.

REFERENCES

[1] K.S. Cline, 2009, The Secrets of the Mathematical Contest in Modelling, v1.8, 22 pages.

[2]http://faculty.washington.edu/hqian/amath4-523/Murray-Math-Biol-ch3.pdf, Models for interacting populations.