SCUDEM IV 2019

PROBLEM CHOSEN: C

CHEMICAL ESPIONAGE

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SUMMARY

With a conflict so to say between butterflies and wasps, to settle the scores is our aim. A number of options throw themselves at us: kill the wasps and save the butterflies seems to be a good idea but puts the ecosystem in a state of imbalance; let things keep going the way they are might be bad for one of the two (namely the butterflies) etc.

Here comes our model, to set a balance between these two players in the ecosystem. So we attempt to set forth how best to manage the interaction and look at what happens in the long run.

INTRODUCTION

Our model takes on an issue of particular species of wasps and butterflies. In a bid to reproduce, the female butterflies produce chemical signals to attract males and of course more than one male gets attracted. The dominant male, as the case may be, finds a way to dissuading or masking other males by producing another signal called the anti aphrodisiacs. This then causes another issue, the wasps, they are sensitive to the anti aphrodisiacs and their larvae if eggs are laid with the butterflies' feed on that of the butterflies.

THE MODEL

In our work, we are combine two models, namely; the logistic growth model and the predator-prey model. the solution we obtain are in this fashion. We demonstrated our models in terms of the following variables;

Q: quantity of chemical signals produced per female butterfly.

A: response of male butterflies to the chemical signal.

$P(t)$: population of butterflies with its relation to time.

$W(t)$: population of wasps with its relation to time.

β: the proportion of butterflies that are males with $0 < \beta < 1$.

k: carrying capacity

c_p and c_w: intrinsic growth rates for butterflies and wasps respectively.

MODEL 1

$$Q(t) = Q_0 - AP_1(t) \tag{1}$$

$$Q(t) \leq Q_0, \qquad Q = Q(P(t))$$

Q_0 is the initial quantity of the chemical signals and Q_t is the quantity of the chemical signals in the environment at a time t. We take an assumption that response to the chemical signals reduces the quantity of signals in the environment.

MODEL 2

We forth a logistic growth model in a prey-predator pattern as follows with the explanation below.

$$\frac{dP}{dt} = c_p P(t)[k - P(t)]$$

in the absence of the wasp population, we favour a logistic model over that of the exponential because the environment and other factors enabling growth are not unlimited and perfect in nature. Bringing in the wasps population and the influence they have, the model has this new form with;

$$\frac{dP}{dt} = c_p P(t)[k - P(t)] - A^* \beta P(t) W(t) \tag{2}$$

$$A^* = \frac{A}{k}, \qquad k > 0.$$

A^* is the *anti aphrodisiacs* density.

We take this since there is an emergence of the *anti aphrodisiacs* that brings on the wasps and the wasps have a way of inhibiting population growth for the butterflies.

This we call the model for the prey(butterflies).

MODEL 3

Let's consider a case where we have all wasps and no butterflies, it turns out as;

$$\frac{dW}{dt} = c_w W(t)[k - W(t)]$$

k still has the same definition. We retain our approach of the logistic model. The presence of the *anti aphrodisiacs* makes it even better

$$\frac{dW}{dt} = c_w W(t)[k - W(t)] + \mu \beta P(t) W(t) \qquad \mu > 0. \tag{3}$$

This we call the model for the predator(wasps).

So far, we have assumed a logistic growth approach to deal with other realistic factors not necessarily mentioned like death, limited nutrition and so on.

NOTE: k has been factored out on both logistic growth models

SOLUTIONS

P can be expressed in terms of W as

$$\frac{dP}{dW} = \frac{c_p P[k - P] - A^* \beta P W}{c_w W[k - W] + \mu\beta P W}$$

from which we can infer that

$$P \neq \frac{c_w}{\mu\beta}[k - W] \text{ and } P < \infty$$

must both be fulfilled for the equation to be defined i.e for a change in population to occur.

Next, we consider the steady-state solutions for P^* and W^* for $P(t)$ and $W(t)$ respectively, from (2) and (3).

$$\frac{dP}{dt} = c_p P(t)[k - P(t)] - A^* \beta P(t) W(t) = 0 \tag{4}$$

and

$$\frac{dW}{dt} = c_w W(t)[k - W(t)] + \mu\beta P(t) W(t) = 0 \tag{5}$$

From (5), we have

$$W = k + \frac{\mu\beta}{c_w} P \quad \text{or} \quad 0$$

which can be substituted into (4), to obtain P as;

$$P^* = \frac{c_p c_w - A^* \beta c_w}{c_p c_w + A^* \mu\beta^2} k \quad \text{or} \quad k \quad \text{or} \quad 0$$

and by substitution, W as;

$$W^* = [1 + \mu\beta \frac{c_p - A^* \beta}{c_p c_w + A^* \mu\beta^2}] k \quad \text{or} \quad [1 + \frac{\mu\beta}{c_w}] k \quad \text{or} \quad k$$

CONCLUSION

The best balance for this system is when (P, W) is

$$(\frac{c_p c_w - A^* \beta c_w}{c_p c_w + A^* \mu\beta^2} k,\ [1 + \mu\beta \frac{c_p - A^* \beta}{c_p c_w + A^* \mu\beta^2}] k)$$

With the equilibra or steady-state solution we have obtained from our models, we can predict that on the long run there would be more wasps than butterflies but with both insects thriving well.

REFERENCES

[1] K.S. Cline, 2009, The Secrets of the Mathematical Contest in Modelling, v1.8, 22 pages.

[2] http://faculty.washington.edu/hqian/amath4-523/Murray-Math-Biol-ch3.pdf, Models for interacting populations.

ADDITIONAL ISSUES

[1] Assume the population of this predator be E(t) and the predator E feeds both insects at the same rate, we can have;

$$\frac{dP}{dt} = c_p P(t)[k - P(t)] - A^* \beta P(t) W(t) - fE(t) \tag{6}$$

$$\frac{dW}{dt} = c_w W(t)[k - W(t)] + \mu\beta P(t) W(t) - fE(t) \tag{7}$$

where f is the rate of consumption of E and other parameters retain their earlier-defined roles.

[2] The female butterfly should go for the male with the lowest *anti aphrodisiac* propensity. this strategy ensures the best safety of the eggs she would eventually lay.

[3] Not so much,

$$\frac{dP}{dt} = T[c_p P(t)[k - P(t)] - A^* \beta P(t) W(t)]$$

and

$$\frac{dW}{dt} = T[c_w W(t)[k - W(t)] + \mu\beta P(t) W(t)]$$

where T is temperature.