Butterfly-Wasp Model

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Designed by Huisheng Zhu, Ran Tao, and Zeman Li

Coach: Christina Lee

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Overview

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- Assumptions
- Model Design
- Models
- Simulation, Results & Analysis
- Conclusion

Restatement of Our Problem

Female butterfly's pheromone release attracts many males (*P. Brassicae*)

Male butterfly release a chemical signal called "***anti-aphrodisiacs*** " to disrupt other males

Wasps, the butterfly egg parasitoid, **could detect the *A.A.***

When a female butterfly has a higher level of ***A.A.,*** Wasps are more likely to find and eat their eggs

Our assumptions

1. This is an idealized ecosystem focusing solely on wasps and butterflies.

2. All wasps possess the same parasitic strategy

3. Ratio of female *P. brassicae* to male *P. brassicae* is 1:1 and probability of giving birth to a male progeny is 50%.

Model Design

What is Best for the system? What is likely to happen in the long run?

Goal: find α (the amount of a.a. secreted by one male butterfly) to maximize **B** (butterfly population) **in the long run**

Model Design

$$\frac{dB}{dt} = g_B B\left(1 - \frac{B}{K_B}\right) - f_1 BW \qquad (1)$$

$$\frac{dW}{dt} = -d_W W\left(1 + \frac{W}{K_W}\right) + f_2 BW \qquad (2)$$

- The established Lotka-Volterra Model fits well into our simulated situation of the competition between butterflies and wasps
- Based on the classic LV Model, we introduce the influence of anti-aphrodisiacs
- d_W = 0.57 (Southon et al., 2015)

Interactions of Male & Female P. brassicae

$$g_B = g_{B0}(1 + m\alpha)$$

$$g_B = 0.348(1 + 0.5\alpha)$$

- g_{B0}: the natural growth rate of butterflies without a.a. = **0.348 (Southon et al., 2015)**
- m: the effectiveness of a.a. in helping female butterflies lay their eggs
- α: the amount of a.a. secreted by one male butterfly (μg)
- g_B: the growth rate of butterflies with a.a.

Interactions of Butterflies & Parasitic Wasps

$$f_1 = f_{10}(1 + h\alpha) \qquad f_1 = 0.01(1 + 0.5\alpha)$$

$$f_2 = f_{20}(1 + h\alpha) \qquad f_2 = 0.005(1 + 0.5\alpha)$$

- f_{10} : butterflies' natural mortality factor without *a.a.*
- h: the effectiveness of *a.a.* in helping wasps eat the eggs = **0.5 (Fatouros et al., 2005)**
- α: the amount of *a.a.* secreted by one male butterfly (μg)
- f_1: butterflies' mortality factor by supplying eggs to wasps
- f_{20}: wasps' natural growth factor without *a.a.*
- f_2: wasps' growth factor by consuming host butterfly eggs

The Model

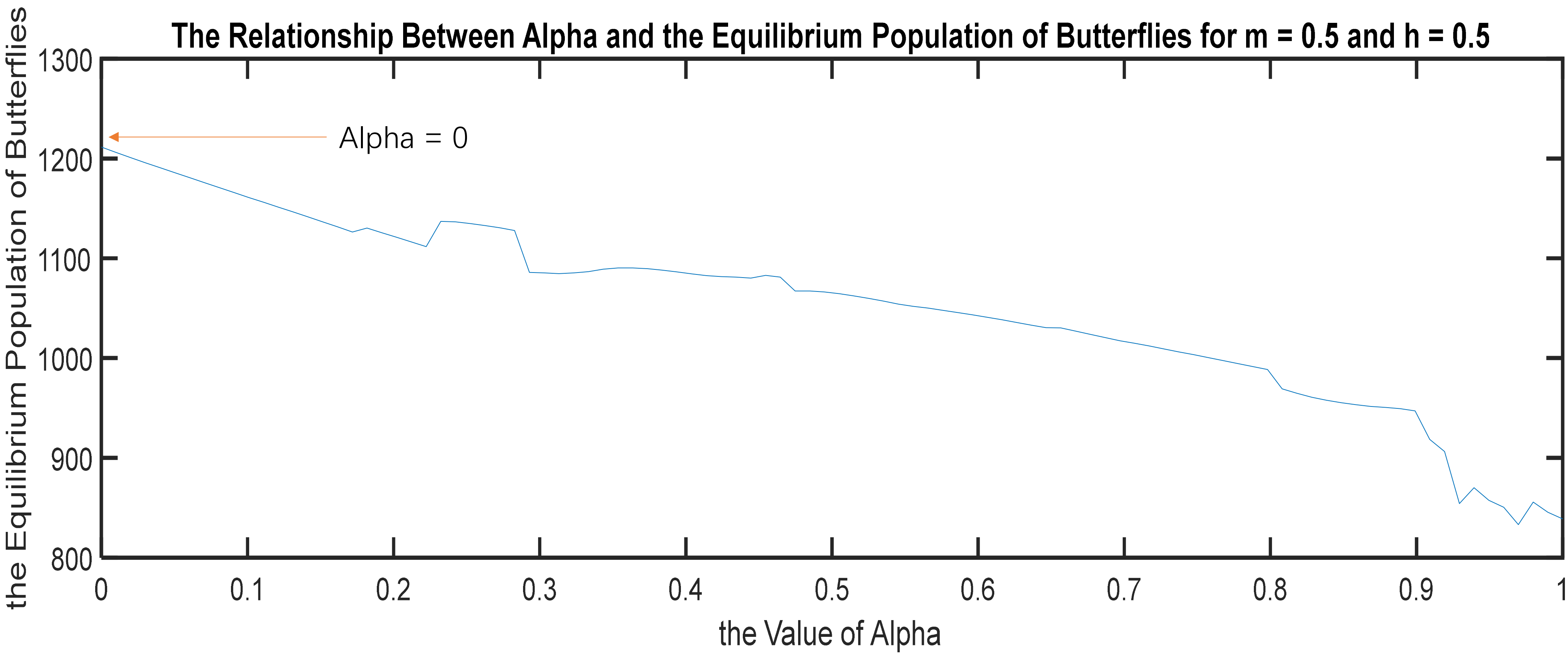
$$\frac{dB}{dt} = 0.348(1 + 0.5\alpha)B\left(1 - \frac{B}{6000}\right) - 0.01(1 + 0.5\alpha)\,BW$$

$$\frac{dW}{dt} = -0.57W\left(1 + \frac{W}{3000}\right) + 0.005(1 + 0.5\alpha)BW$$

Initial conditions

$$B(0) = 2000$$
$$W(0) = 500$$

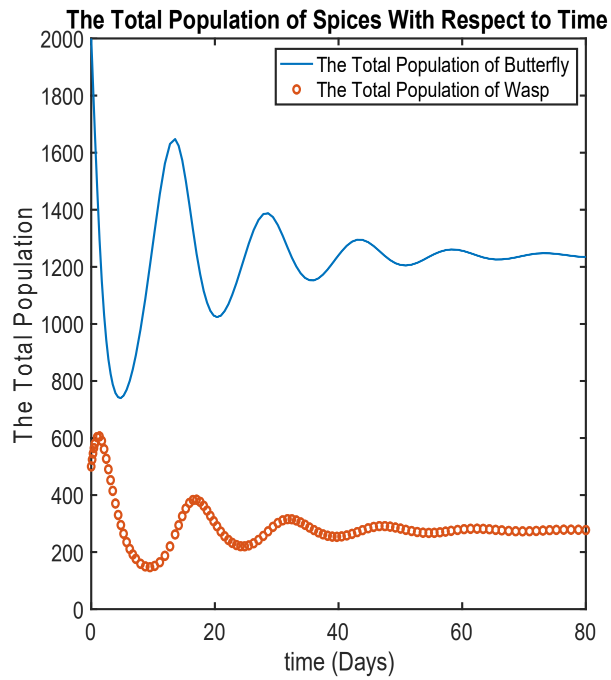
Simulation, Results & Analysis

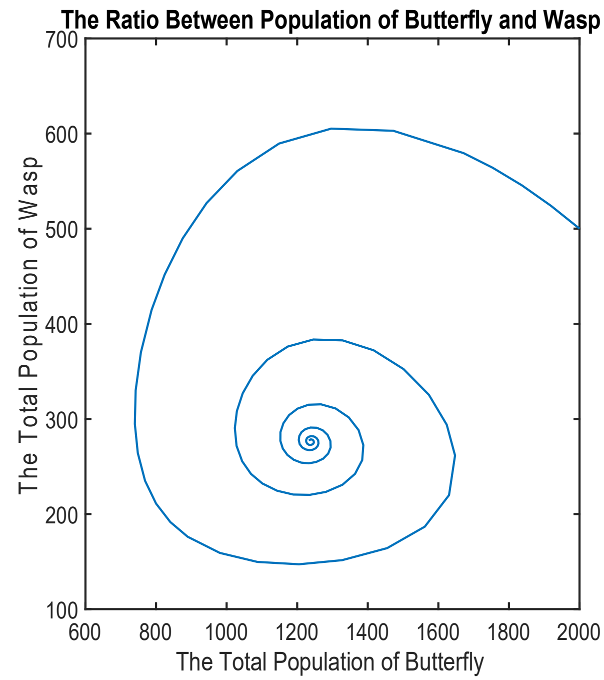


Simulation, Results & Analysis

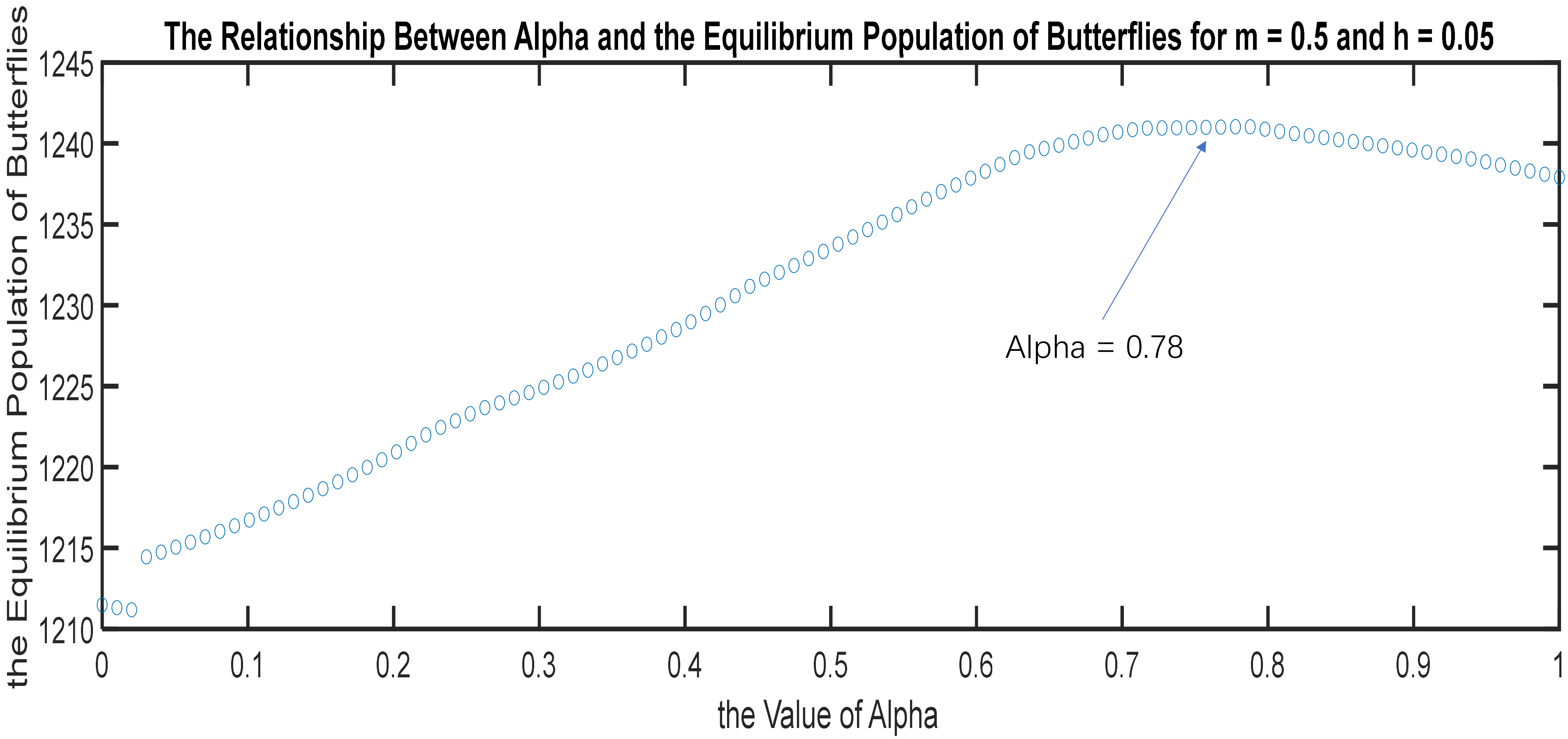
Characteristics of Our Simulation (*h* = 0.5, alpha = 0)

- Butterfly reaches its minimum around Day 5
- As the wasp population remains at a high level, butterfly population starts to recover
- A cyclical repeat
- Equilibium population or fixed point





Simulation, Results & Analysis

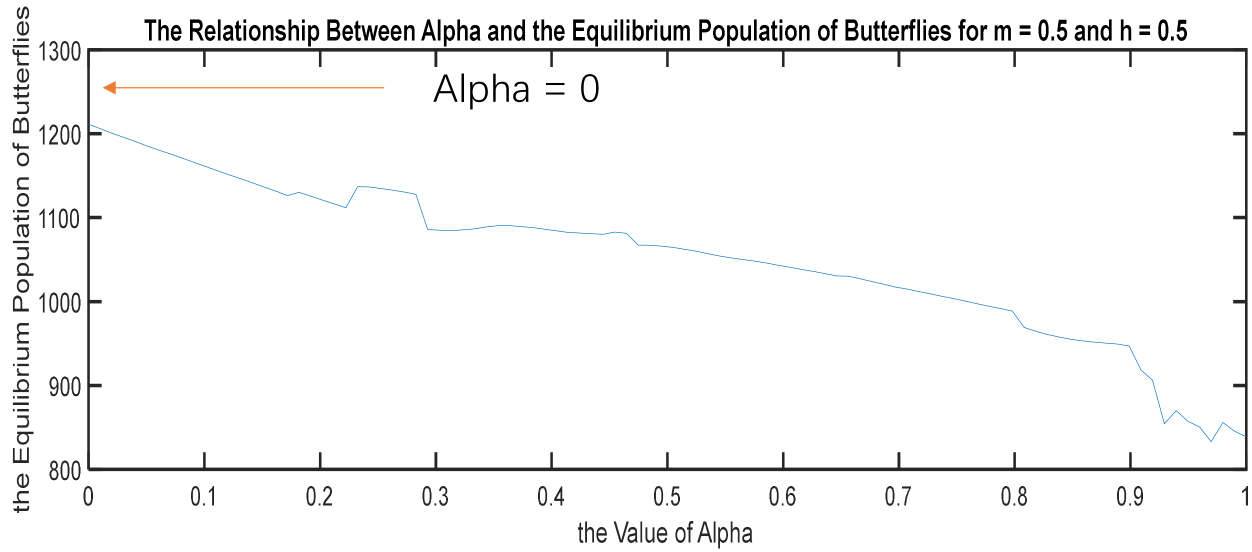


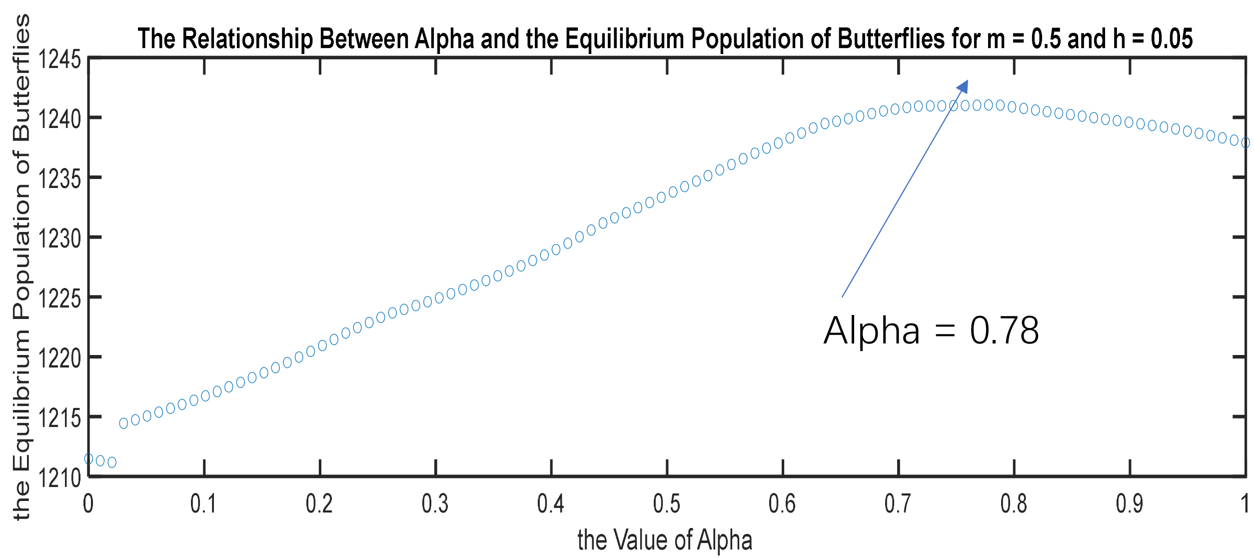
Simulation, Results & Analysis

- Explanation
  - When *h* change from from 0.5 to 0.05, the alpha of maximum equilibrium population change from 0 to 0.78

  - Possible explanation from the viewpoint of evolution

"Interestingly, this behavior is innate in T. brassicae, whereas T. evanescens learns it after one successful ride on a mated female butterfly" (Huigens et al., 2010).





Additional Issues

- **Additional Change 1 - when we introduces a new predator on both butterflies and the wasps?**

In LV models, if we have animals more than two, we could use the following equation:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{\sum_{i=1}^{i=N} \alpha_{ij} x_j}{M_i}\right)$$

where r_i denotes the birth rate of animal x_i, α_{ij} denotes the relative impact of animal x_j on the growth of anima x_i, and M_i denotes the carry capacity of animal x_i

In this simulation, we get the equations as follows: we assume the third animal is bird and denotes its population with D, the natural death rate of the birds is d_d, and the ecosystem's carrying capacity of birds is K_d. The bird's impact on butterflies is f_3 and the bird's impact on wasps' impact is f_4.

$$\frac{dB}{dt} = g_B B\left(1 - \frac{B}{K_B}\right) - f_1 BW - f_3 DB \ (1)$$

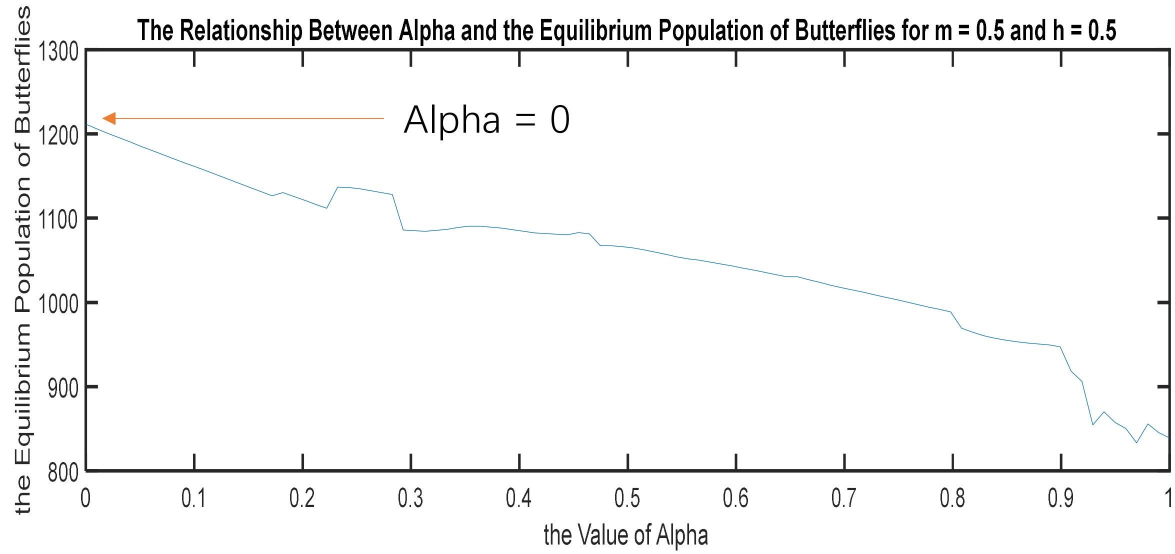
$$\frac{dW}{dt} = -d_W W\left(1 + \frac{W}{K_W}\right) + f_2 BW - f_4 DW \ (2)$$

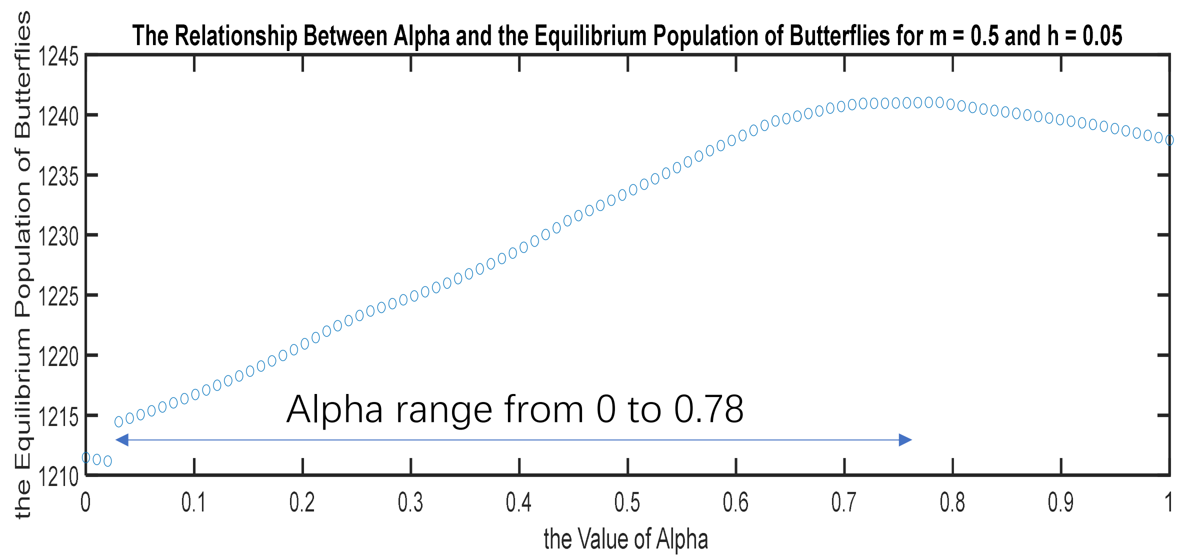
$$\frac{dD}{dt} = -d_D D\left(1 + \frac{D}{K_D}\right) + f_3 DB + f_4 DW (3)$$

Additional Issues

- **Additional Change 2 - when we introduces a new predator on both butterflies and the wasps?**

- Female butterfly's strategy depends on the trade-offs of releasing anti-aphrodisiac.
- Under condition that h = 0.5, female prefers to mate with males that have low propensity to releasing a.a.
- Under condition that h = 0.05, when alpha range from 0 to 0.78, female prefers to mate with the male that higher propensity to release a.a.





Additional Issues

- **Additional Change 3 – what if effectiveness of a.a. depends on time of day?**

$$\frac{dB}{dt} = 0.348(1 + m(t)\alpha)B\left(1 - \frac{B}{6000}\right) - 0.01(1 + h(t)\alpha)\, BW$$

$$\frac{dW}{dt} = -0.57W\left(1 + \frac{W}{3000}\right) + 0.005(1 + h(t)\alpha)BW$$

Initial conditions

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Reference

- Huigens, M. E., Woelke, J. B., Pashalidou, F. G., Bukovinszky, T., Smid, H. M., & Fatouros, N. E. (2010). Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps. *Behavioral Ecology*, *21*(3), 470-478.

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