SCUDEM IV 2019

Problem C: Chemical Espionage

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Date: November 8, 2019

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Can you smell the love?

- Large cabbage white butterflies (*Pieris brassicae)*
- Pheromones to attract mates
- Males produce anti-aphrodisiac
- Anti-aphrodisiacs reduce attractiveness of female
- Female can oviposit in ideal location

Image 1: Large cabbage white butterflies (*P. brassicae)*

Chemical Espionage

- Parasitic Wasp (*Trichogramma brassicae)*
- Exploits anti-aphrodisiac
- Hitchhikes on large cabbage white butterflies
- Leads to oviposition of eggs
- Parasitize butterfly's eggs

Image 2:Parasitic Wasp (*T. brassicae)* parasitizing egg

Statement of Problem

- Develop a mathematical model for the interactions of the male and female butterflies as well as the parasitic wasps with the use of anti-aphrodisiac.

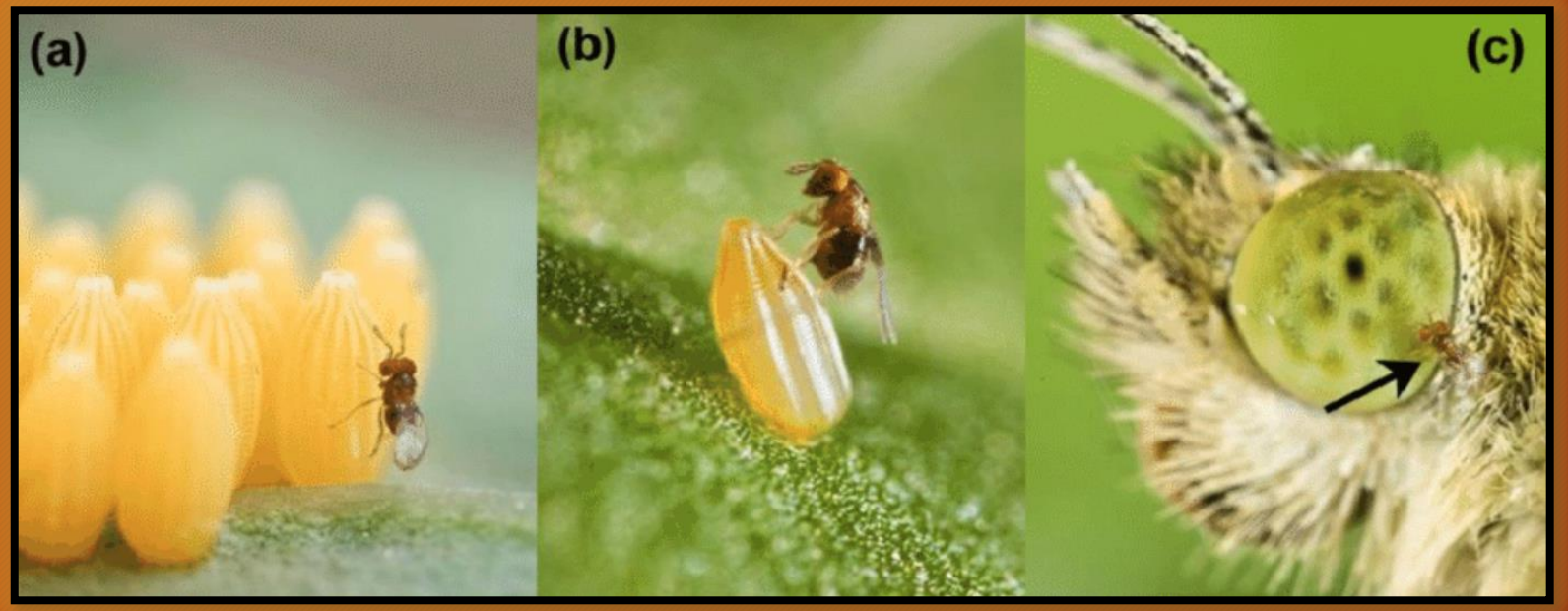


Image 3: Parasitic wasp exploiting anti-aphrodisiac

Assumptions

1. Butterfly eggs are the only resource for the parasitic wasp.
2. Population of males does not have an impact on the number of fertile female butterflies.
3. Logistic growth for butterflies.
4. The eggs parasitized is proportional to the number of wasps.
5. Continuous rate of new butterflies and eggs being made year-round.

Model of Interaction

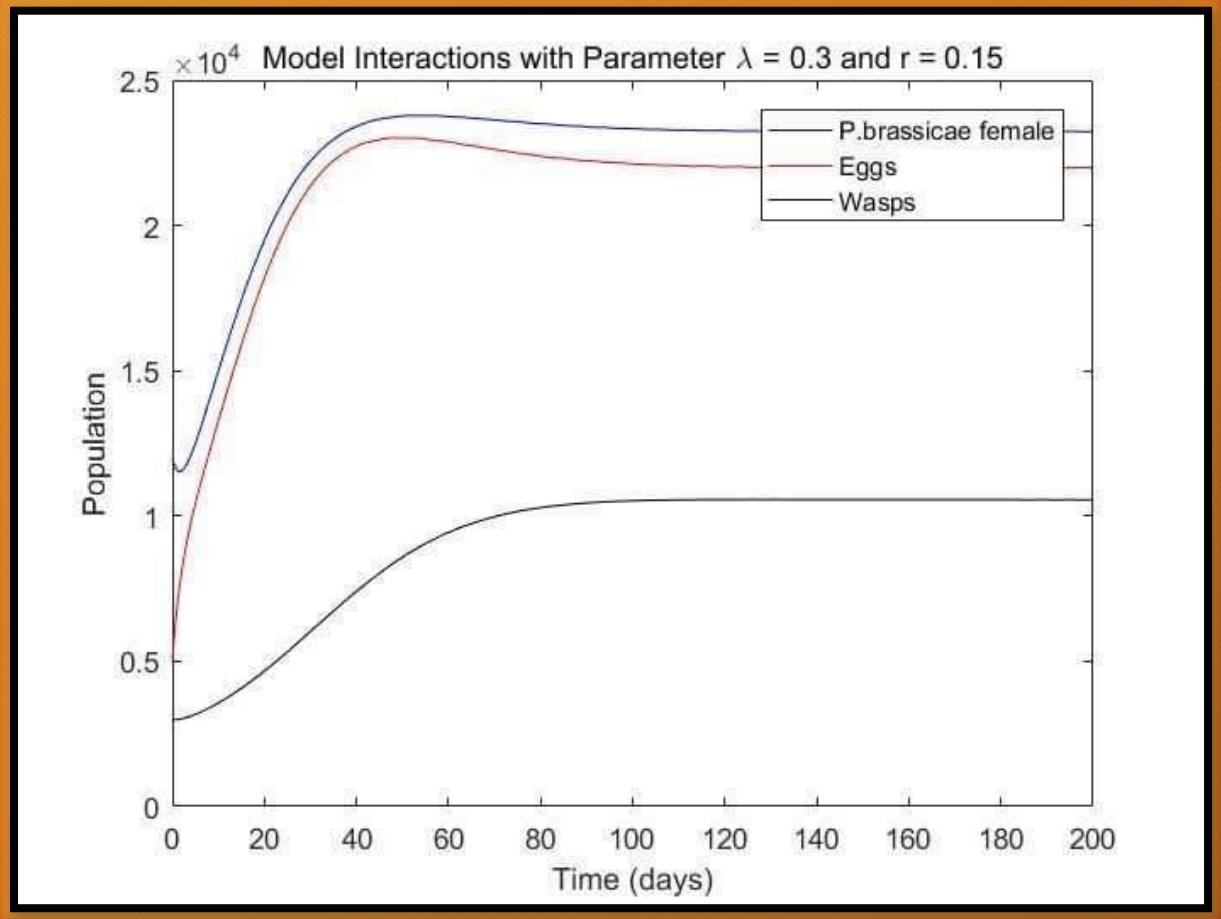
1. $\frac{dB}{dt} = \lambda E(1 - \frac{B}{K}) - k_B B$

2. $\frac{dE}{dt} = \sigma B - \lambda E - r\frac{EW}{E+W}$

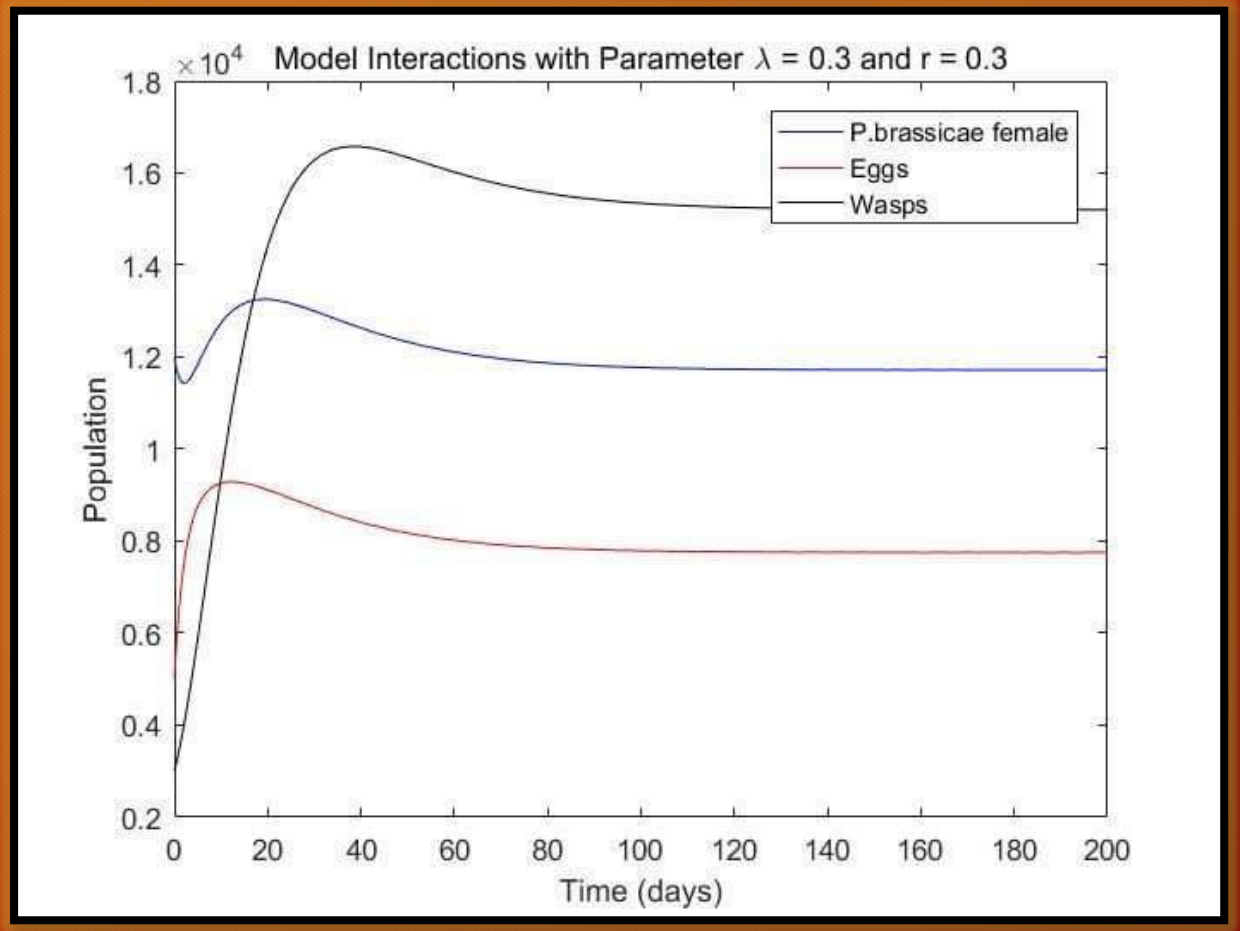
3. $\frac{dW}{dt} = jr\frac{EW}{E+W} - k_W W$

- $B = number\ of\ female\ butterflies$
- $E = butterfly\ eggs$
- $W = wasps$
- $K = carrying\ capacity\ of\ female\ butterflies$
- $j = wasps\ larvae\ scaling\ factor$
- $\lambda = percent\ of\ eggs\ hatched\ into\ female\ butterflies$
- $\sigma = fertility\ rate\ of\ female\ butterflies$
- $r = number\ of\ parasitized\ eggs$
- $k_B = death\ rate\ of\ female\ butterflies$
- $k_W = death\ rate\ of\ wasps$

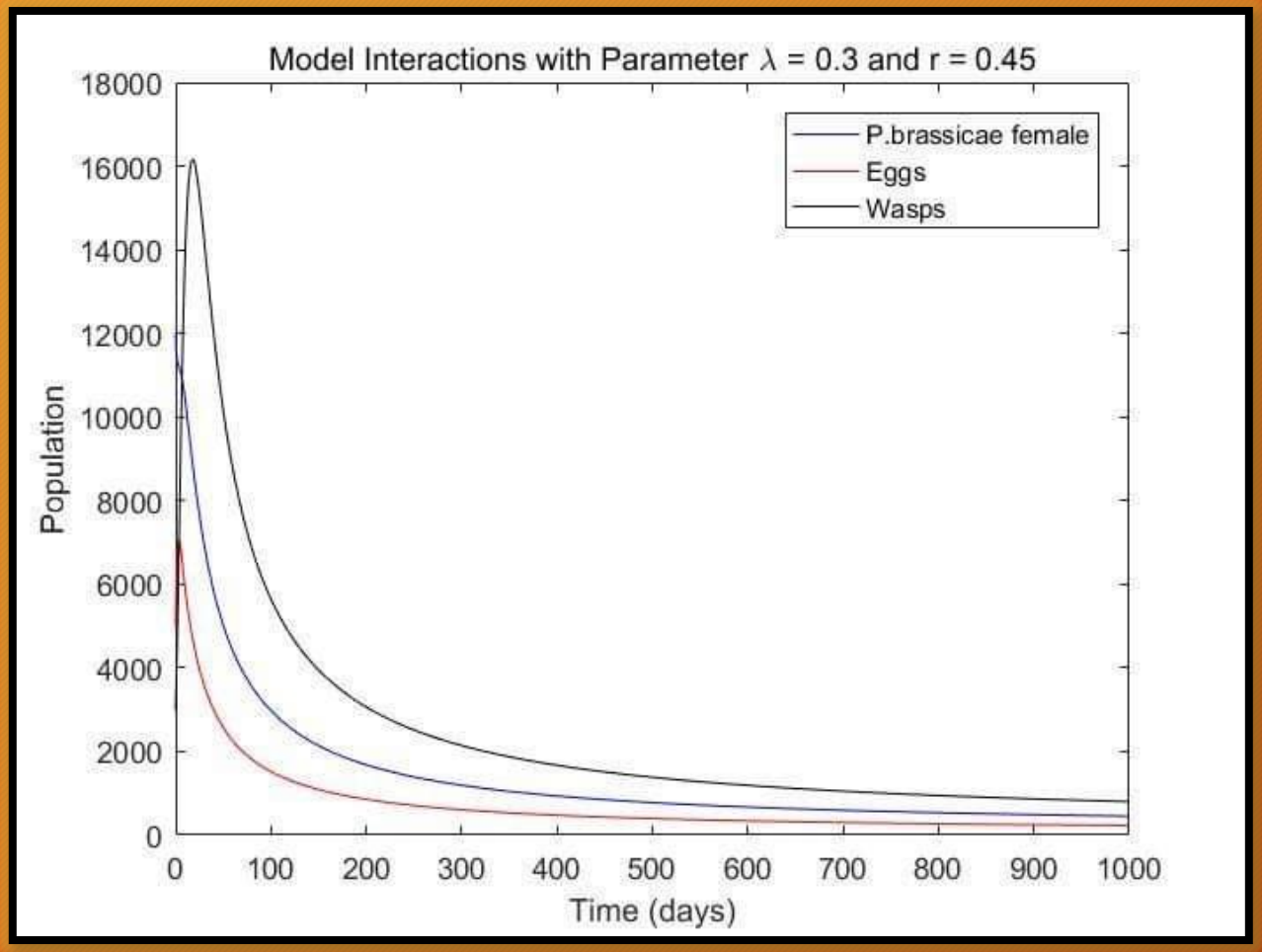
Results



Graph 1

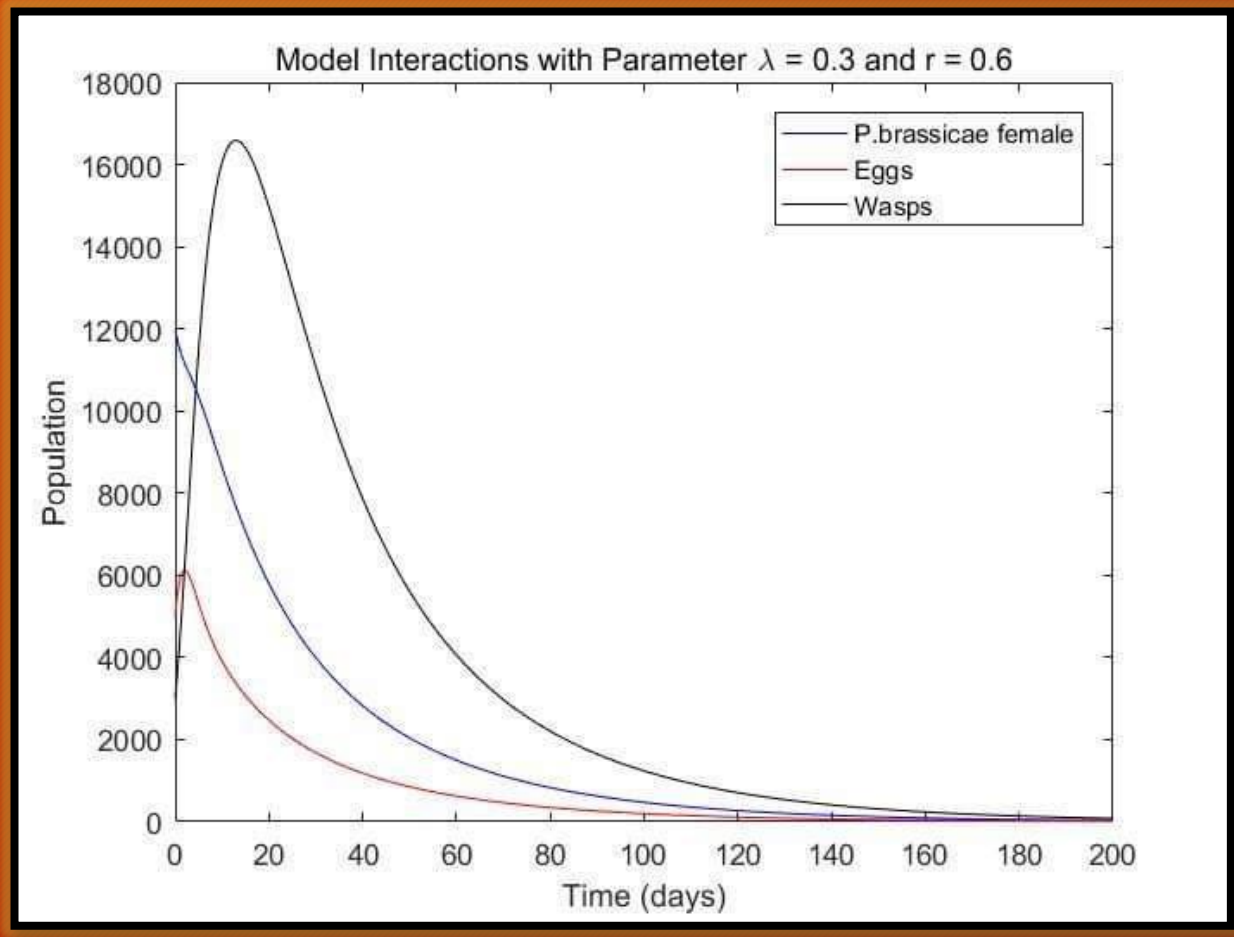


Graph 2



Graph 3

Graph 4



Complication:

- Bird which preys on both the butterflies and the parasitic wasps. Proposed changes to our model:
  - Assume bird population is constant.
  - $\frac{dB}{dt} = \lambda E\left(1 - \frac{B}{K}\right) - k_B B - h_b \frac{FB}{B+F}$
  - $\frac{dW}{dt} = jr\frac{EW}{E+W} - k_W W - h_W \frac{FW}{W+F}$

Conclusion

- Anti-aphrodisiac induced egg stealing rate > beneficial rate of growth for the butterflies
  - ➢butterfly population should go extinct

- Anti-aphrodisiac induced egg stealing rate < beneficial growth rate for the butterflies
  - ➢populations of the butterflies and the wasps will reach stable, nonzero levels

Resources

[1] Image 1: https://konlinejobs.com/pieridae-butterflies-facts-classification-images-information/#Family_PieridaeButterfliesIntroduction

[2] Image 2: https://subsites.wur.nl/en/show/RP12-Monitoring-pre-and-postrelease-diversity-in-local-parasitoid-populations.htm

[3] Image 3: https://www.researchgate.net/figure/Indirect-defences-Parasitoid-wasps-are-attracted-by-B-nigra-as-an-indirect-defence_fig6_317427275