Chemical Sabotage

Question C

Zach Kelly, Ryan McDaniel, Ben Wilfong

Dr. Joseph Eichholz

Rose-Hulman Institute of Technology

Problem Description

- A certain species of butterflies (*Pieris brassicae*) and wasps (*Trichogramma brassicae* and *Trichogramma evanescens*) have a parasitic relationship in which the wasps lay their eggs in the butterfly eggs.
- When the butterflies mate, the male releases an anti-aphrodisiac that wards off other males, allowing the females to lay eggs without being harassed.
- The anti-aphrodisiac also attracts the parasitic wasps.
- Our task is to model this interaction and find the optimal amount of anti-aphrodisiac to release.

Driving Assumptions

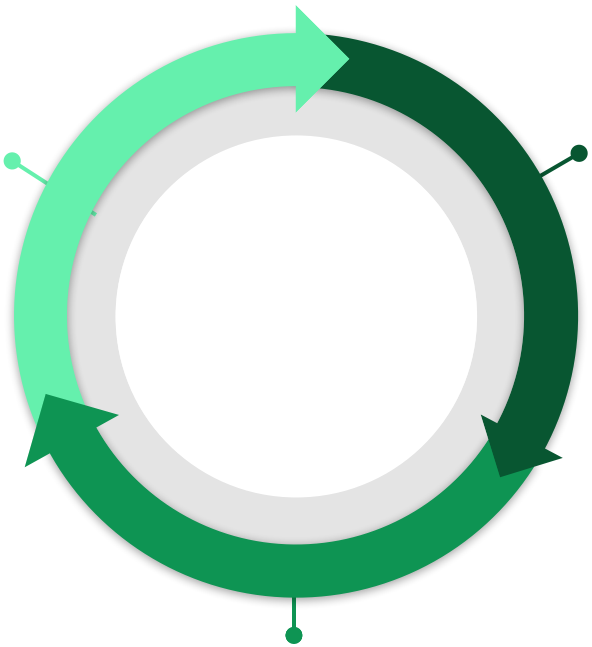
- To simplify the model, we made many assumptions:
  - The populations of both butterflies and wasps are split evenly between males and females.
  - Both species of wasps interact with the butterflies in similar enough ways that they can be treated as one large wasp population.
  - Fertilized females of both wasps and butterflies are always ineligible for mating, while unfertilized females and males are always eligible.
    - Fertilized females become unfertilized upon laying their eggs.
  - The environment has limited resources, so both wasps and butterflies have a carrying capacity.

Driving Assumptions, Continued

- To simplify the model, we made many assumptions:
  - Butterflies lay eggs, wasps lay eggs, and wasps find nests all at the same time.
  - If a wasps finds a butterfly nest, no butterfly eggs from that nest survive.
  - Butterflies and wasps always die after a certain amount of time.
  - Egg batches always contain the same number of eggs, and eggs always hatch after the same amount of time.
  - Butterflies become eligible for mating immediately after maturing.

Basic Model Idea

Eggs hatch and animals grow to maturity during a time.



Adults of a species mate to fertilize females of the population. They die after a set lifespan.

Fertilized females will lay their eggs. Nest control takes into effect here.

Butterfly and Wasp Fertilization

Incorporates a carrying capacity: $\frac{d}{dt}X(t) = \frac{K - X(t)}{K} X(t)$

Other traits:

Fertilization Rate	Male (X_m)	Female (X_{fem})	Fertilized Females (X_{fert})	Anti-aphrodisiac Release (A)
0	0			
max		equal	equal	
0 for Butterflies				0

$$\frac{d}{dt}B_{fert}(t) = c_1 A \frac{K_b - B(t)}{K_b} \sqrt{(B_{fem}(t) - B_{fert}(t))\, B_m(t)} - c_1 A \frac{K_b - B(t - T_b)}{K_b} \sqrt{(B_{fem}(t - T_b) - B_{fert}(t - T_b))\, B_m(t - T_b)}$$

$$\frac{d}{dt}W_{fert}(t) = c_2 \frac{K_w - W(t)}{K_w} \sqrt{(W_{fem}(t) - W_{fert}(t))\, W_m(t)} - c_2 \frac{K_w - W(t - T_w)}{K_w} \sqrt{(W_{fem}(t - T_w) - W_{fert}(t - T_w))\, W_m(t - T_w)}$$

Parasitism by Wasps (Nest Hijacking)

Desirable Traits:

Hijacking Rate H(t)	B_{fert}	W_{fert}	A
0	0		
0		0	
W_{fert}	Infinite		
B_{fert}		Infinite	
0			0

$$H(x_1, x_2) = \frac{2}{x_1^{-1} + x_2^{-1}}$$

$$H(t) = \frac{c_3 A}{\left(B_{fert}(t - T_b)\right)^{-1} + A\, W_{fert}(t - T_w)^{-1}}$$

Supporting Cast

Eggs:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}E_b(t) = L_b\left(\frac{\mathrm{d}}{\mathrm{d}\,t}B_{fert}(t) - H(t)\right) - L_b\left(\frac{\mathrm{d}}{\mathrm{d}\,t}B_{fert}\left(t - M_b\right) - H\left(t - M_b\right)\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}\,t}E_w(t) = L_w H(t) - L_w H\left(t - M_w\right)$$

Population sizes:

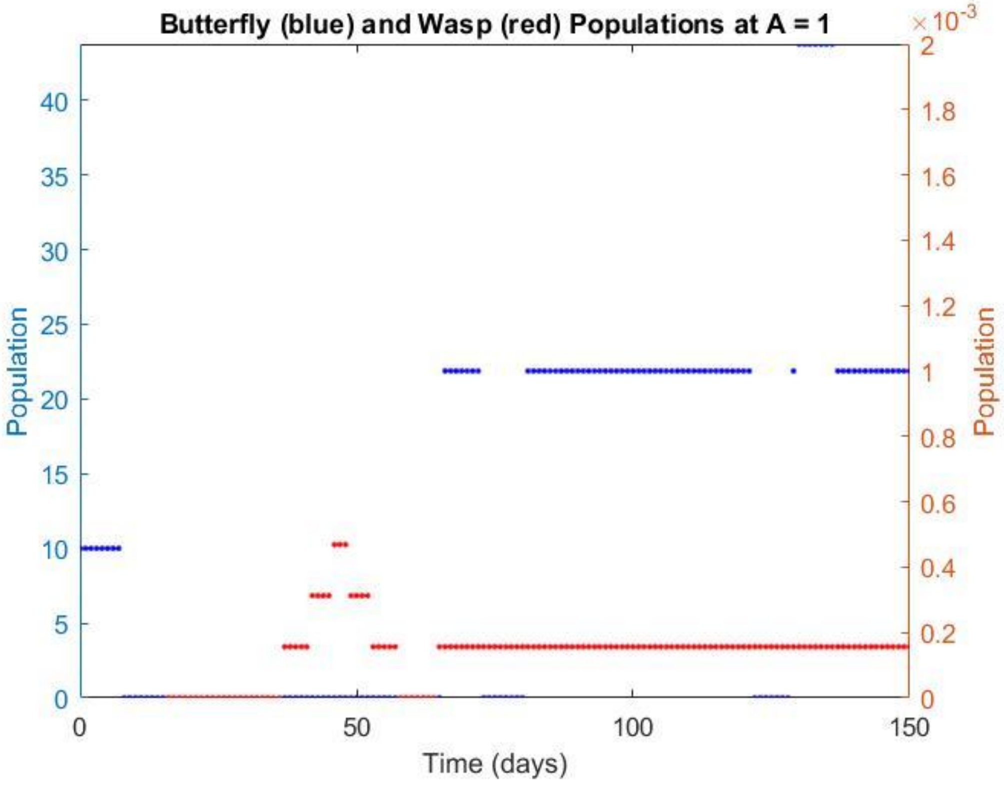
$$\frac{\mathrm{d}}{\mathrm{d}\,t}X(t) = \frac{\mathrm{d}}{\mathrm{d}\,t}E_x\left(t - H_x\right) - \frac{\mathrm{d}}{\mathrm{d}\,t}E_x\left(t - H_x - \mathrm{D}_x\right)$$

$$X_m(t) = X_{fem}(t) = \frac{1}{2}X(t)$$

Results: A steady state solution is found.

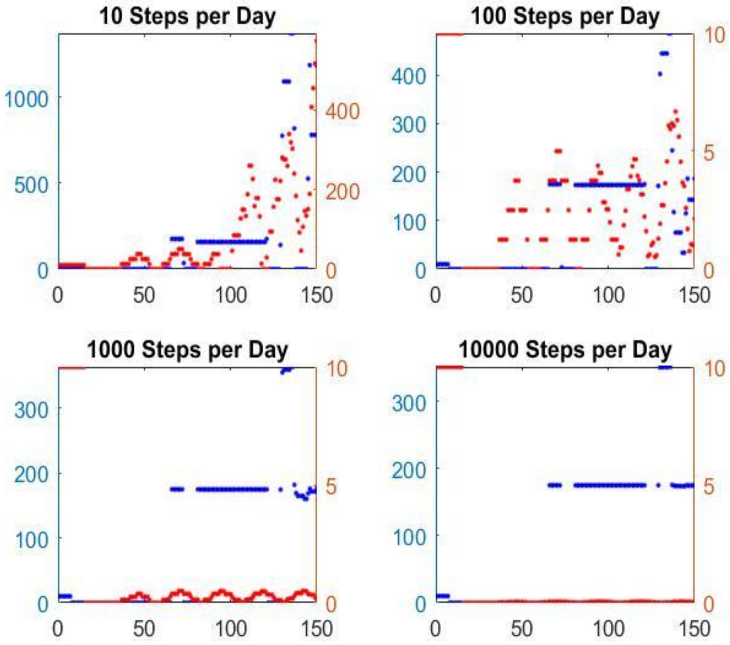
Simulated for 150 Days

Starting populations are 10 each



The model is nonlinear.

- Solutions were found using Euler's method



Other Issues

- Anti-Aphrodisiac Release
- Wasp population
- Butterfly population discontinuity