Chemical Espionage Solution

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Introduction to the problem

- Population Model
- Seperate Male and Female butterfly for specific solution
- Relationship between butterflies and wasps
- anti-aphrodisiac as key factor

Assumptions

1) Every time each female interacts with a male, a certain amount of anti-aphrodisiacs will be released by the male
2) The number of males and females butterflies are about the same number
3) When the ecosystem reaches its best balance, it contains similar number of butterflies and wasps
4) Butterflies and wasps have predator-prey relationship, and wasps population depends on butterflies as resources to survive

Notations

Female Butterfly Population at t years	$\mathrm{F}(t)$
Male Butterfly Population at t years	$\mathrm{M}(t)$
Butterfly Population at t years	$B(t) = \mathrm{F}(t) + \mathrm{M}(t)$
Wasp Population at t years	$\mathrm{W}(t)$
Butterfly Birth Rate	$\alpha + \beta B(t) - \gamma B(t)W(t)$
Butterfly Death Rate	$x + yB(t)$
Wasp Birth Rate	a
Wasp Death Rate	$b - cB(t)W(t)$

Model

Population model of Butterfly Population by time:

$$\frac{dB}{dt} = \left(\alpha + \beta F(t) - \gamma F(t)W(t)\right)(F(t) + M(t)) - \left(x + y\left(F(t) + M(t)\right)\right)\left(F(t) + M(t)\right)$$

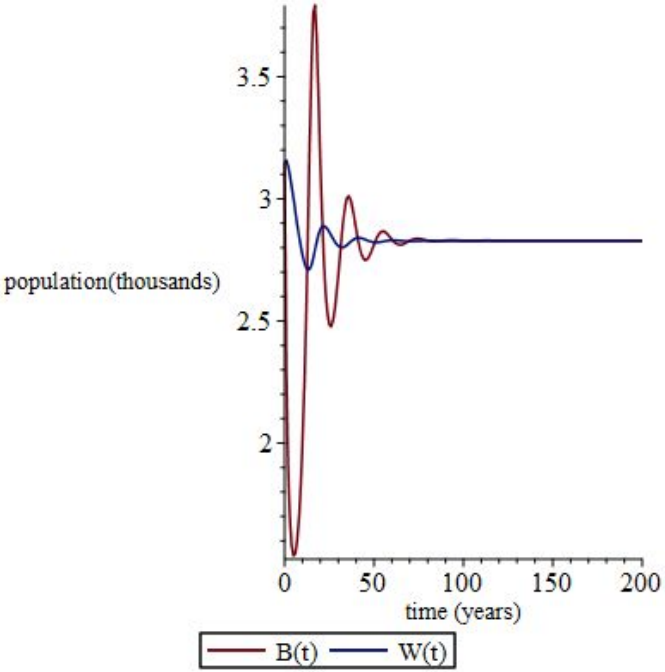
Population model of Wasps Population by time:

$$\frac{dW}{dt} = aW(t) - \left(b - cF(t)W(t)\right)W(t)$$

Simulation

- Differential Equations with constants and variables
- Assume high birth rate of butterflies
- Assume natural birth rate of wasps lower than death rate

Graphs and Simulation Results



B(t)= butterfly population

W(t)= wasp population

$$\frac{dW}{dt} = aW(t) - \big(b - cB(t)W(t)\big)W(t)$$

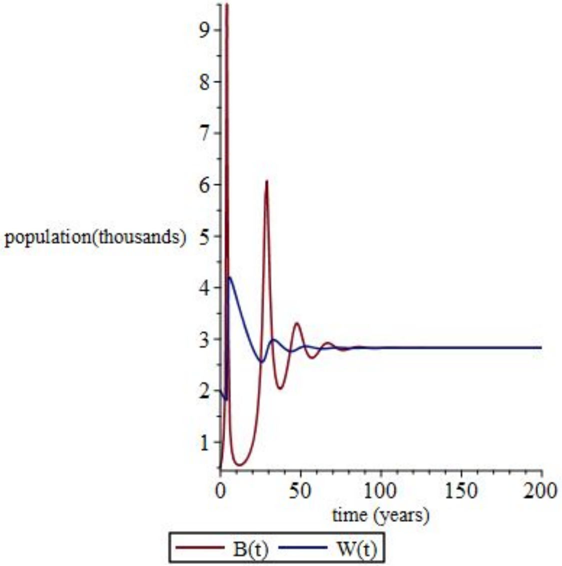
$$\frac{dB}{dt} = \big(\alpha + \beta B(t) - \gamma B(t)W(t)\big)B(t) - \big(x + yB(t)\big)B(t)$$

$\alpha = 0.2 , \beta = 0.9 , \gamma = 0.3 , x = 0.001 , y = 0.01$

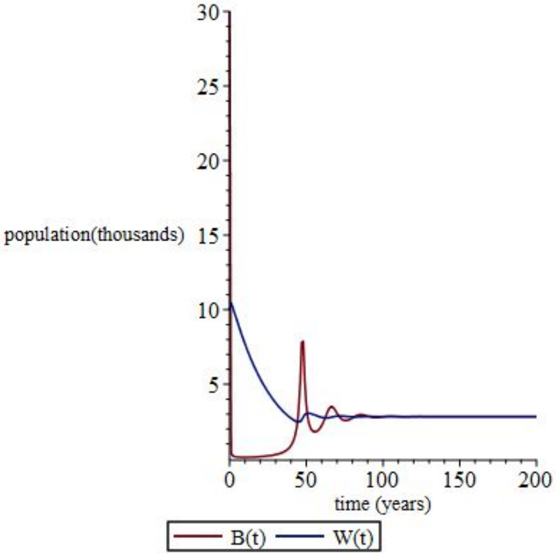
$a = 0.01 , b = 0.05 , c = 0.005$

Graph with modified initial conditions

In these graphs, the initial conditions are modified, but in the long run, the output of both population remains the same.

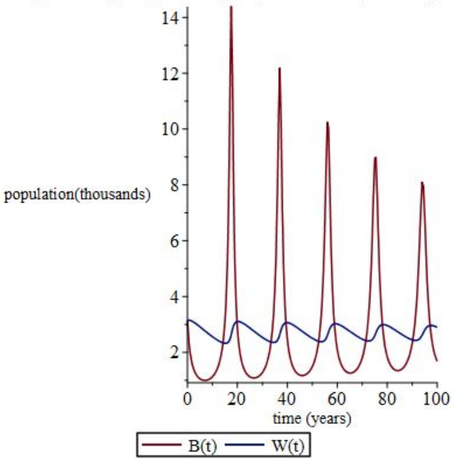


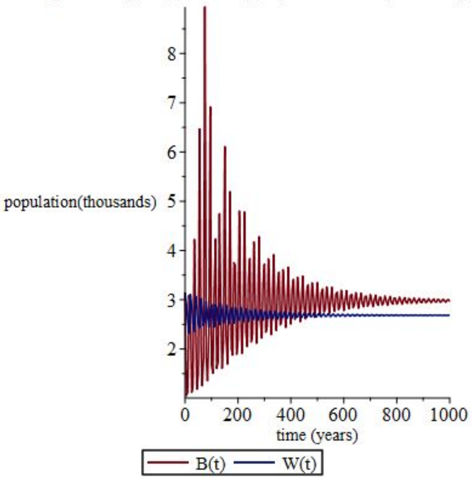
B(0)=0.5, W(0) = 2



B(0)=30, W(0)=10

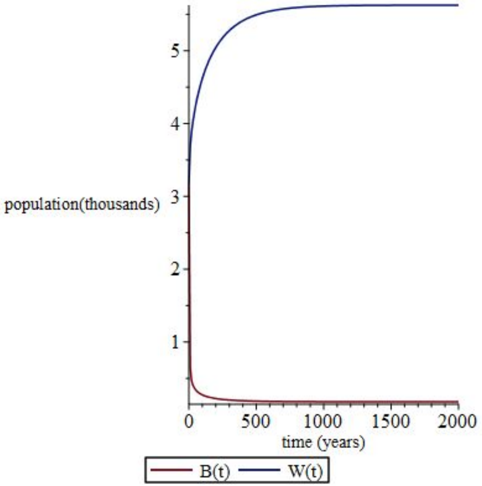
Effect of constants changes in diff(B,t)

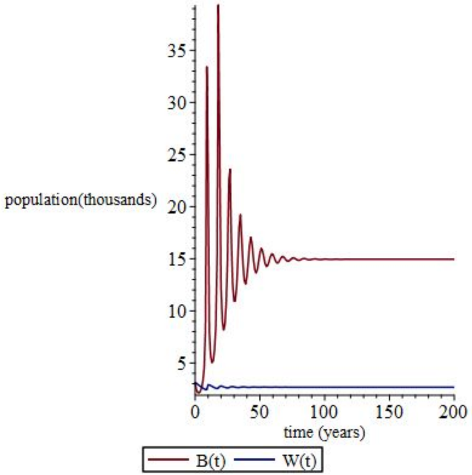




Smaller difference between natural death rate and natural birth rate lead to more drastic change of butterflies and wasps population periodically, but in the long run the system approaches equilibrium.

Effect of constants changes in diff(W,t)

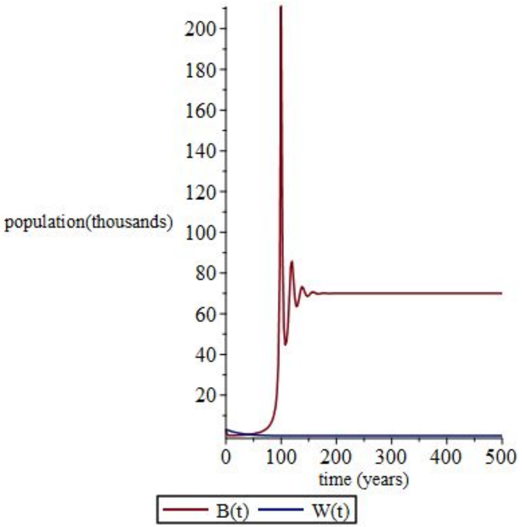




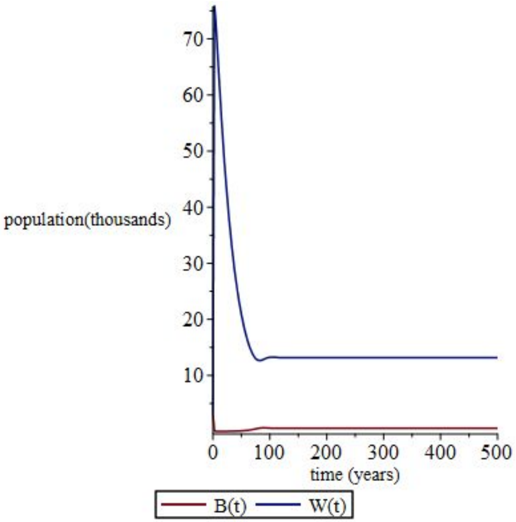
Lower death rate

Interations with butterflies multiplied by smaller constant

Effect of constants changes in diff(B,t)



when β decreases (0.05)



when γ decreases (0.1)

Conclusion

- Both population of wasps and butterflies will become constant in the long run.
- Final population unrelated to initial conditions
- Higher butterflies birth rate due to the positive effect of anti-aphrodisiac leads to less butterflies and more wasps in the long run
- Lower birth rate due to anti-aphrodisiac’s attraction to wasps leads to more butterflies and less wasps in the long run

Thank you