

SCUDEM IV 2019 - Problem C: Chemical Espionage

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Introduction To Problem C

- There is a ecological system which contains a species butterfly and a species of wasps.
- The male butterfly population use anti-aphrodisiacs when mating with females because the anti-aphrodisiacs deter other males
- Wasps can detect the anti-aphrodisiacs on mated females and ride them to the location where the female lays her eggs.
- Wasps then lay their own eggs, and when the wasp larvae hatch they eat the butterfly eggs

Assumptions

- Both butterfly and wasp populations do not decrease as a result of creatures from or to outside populations
- There is enough food to infinitely sustain the butterfly population
- Every egg that the wasp parasitizes becomes a wasp embryo
- Every baby butterfly or wasp born will make it to its adult stage
- Female butterflies only need to mate with one male to be fertilized

Our Model

$$\frac{dP}{dt} = \frac{dP_F}{dt} = \frac{dP_M}{dt} = AP_F + BP_M - CP_MP_F - D(AP_F + BP_M)P_MW$$

$$\frac{dW}{dt} = D(AP_F + BP_M)P_MW - EP_FW$$

P = Total population of butterflies ($P_f + P_m$)

P_F = Population of female butterflies

P_M = Population of male butterflies

W = Population of Wasps

A, B, C, D, E are positive constants

Clarification of Model

$$\frac{dW}{dt} = \frac{D(AP_F + BP_M)P_M W}{\text{---}} - \frac{EP_F W}{\text{---}}$$



Growth
Rate of
Wasps

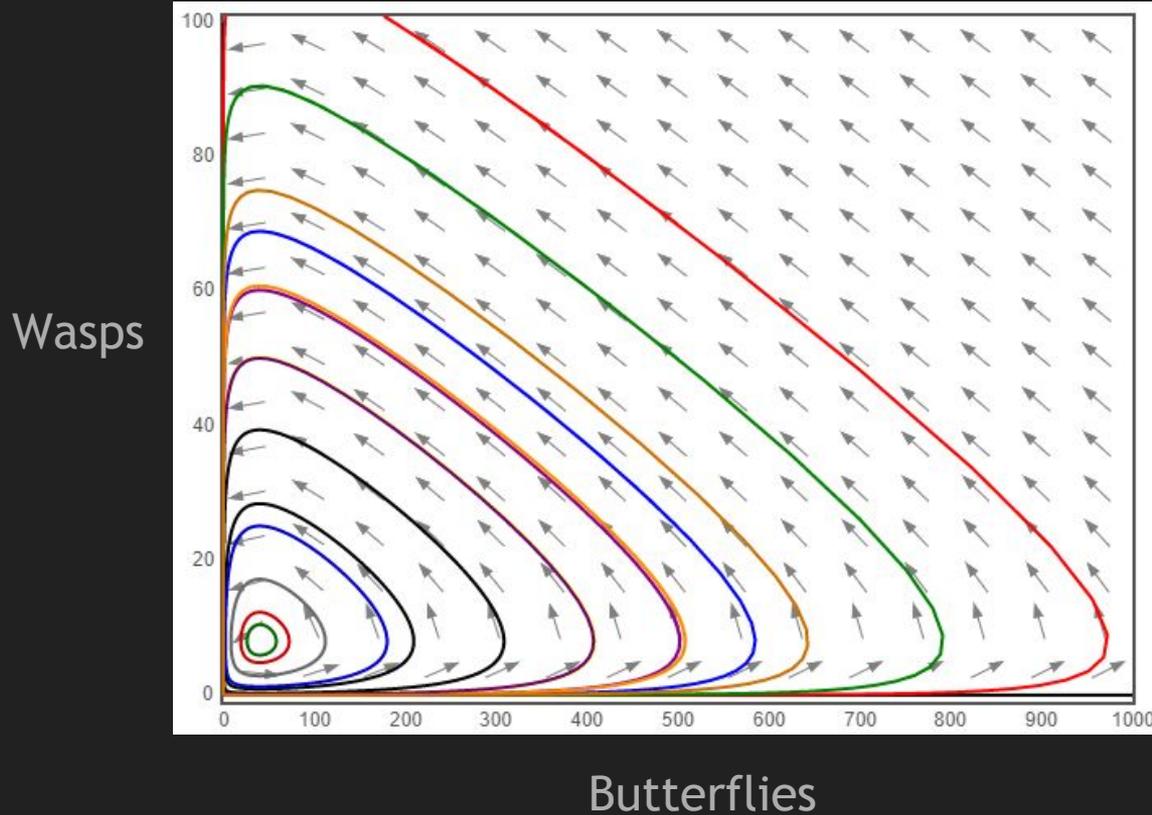


Birth Rate
of
Butterflies



Natural
Death Rate
of Wasps

Examples of Graphs



Model Limits / Possible Improvements

- Does not factor in any outside events that may affect growth rates of butterfly and wasps populations
- Can model the transfer rate of anti-aphrodisiac from male butterfly to female butterfly
- Model does not account for all possibilities for a real system

Additional Issues

1. To accommodate for a predator that eats both the wasps and the butterflies, we would add a third differential equation that models the birds and add terms to the first two equations that accounts for the population eaten by the birds.
2. If the female butterfly does not naturally account for wasp behaviour, it would choose a mate with the most anti-aphrodisiac. However, if the female does account for wasp behaviour, then it would choose a mate with the lowest anti-aphrodisiac
3. If the effectiveness of the anti-aphrodisiac depends on the time day, we can adjust our model by multiplying all BP_M terms by a sinusoidal function $F\sin(Gt+H)$ where F , G , and H are constants.

References

[1] “Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps, “ Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros. Behavioral Ecology. Volume 21, Issue 3, MayJune 2010, Pages 470-478, 11 February 2010. <https://doi.org/10.1093/beheco/arq007>.