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SCUDEM IV 2019 Problems - Overview

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1 Overview

Three questions were posed as part of SCUDEM IV 2019. The first question required teams to model the changes and migrations associated with how people change their personal appearance. The second question required teams to determine how to land and move a small probe on a small asteroid. The third question required teams to explore the population dynamics associated with parasitism when the behavior of the animal being preyed upon impacts the rate of parasitism.

This year the vast majority of teams chose to examine the third question on parasitism. The next most popular choice was the second question, group affinity. The question about micro-gravity received the least attention.

The SCUDEM IV 2019 event included a number of activities, most notably the teams working together to develop a model to gain insight into the question posed. My focus here is on the teams' Executive Summary as well as the slides submitted by the teams from their Presentations. Comments are provided in four sections. The first three focus on each of the problems in turn and in the last section I make some general comments about modeling and presenting results.

2 Problem A: Group Affinity

The question on group affinity was the second most popular question, but the number of teams examining it was much smaller than those who chose to explore the question on parasitism. Despite the smaller number of teams who chose this question, the range of approaches was similar, and the teams brought a great deal of creativity and insight into this question. In this overview of the problem I will first focus on the different types of models employed. Next I will provide some discussion about the basic modeling issues some teams explored. Finally, I will cite some notable aspects of how the teams presented their results.

2.1 Models Used

The teams made use of an impressive range of models. The most common approach was to make use of a logistic model. Another common set of models were based on disease models ranging from a standard SIR model to an SIS model in which people who leave a group potentially can move back into the group at a later time. Other types of models were based on the heat equation while others made used of randomness to mimic how people decide to move between groups.

Underlying most of the models is the idea of breaking up a population into different categories, or compartments, and then modeling the movement between compartments. The primary difference between the models is how to approximate the rate that people move between the groups.

For example, in the case of an SIR model, there are three compartments. The first is the group of Susceptible people who are not part of the group of people who have adopted a particular habit. The second group is the Infected people who have adopted the habit. The final group is made up the people who have Recovered and no longer have the given habit. The basic assumption is that the rate that people move between two groups is proportional to the product of the size of the populations in two groups. The idea is that this product is an indication of the likelihood that two people from two groups will interact and one person will adopt the behaviour from another group.

This kind of model is an effective tool in modeling the way a disease can be shared between people in a community. One difference, though, in this situation is that people do not become immune and can move back and forth between the different groups as well as moving back into a group of they were previously a member. Teams that were able to adapt their model to accommodate this difference had an important insight into the basic behavior of the people in this situation.

Being able to adapt or improve a mathematical model is an important part of the modeling process. Some teams were able to demonstrate how to start with a good model, provide a critical analysis, and then improve their model. This is a relatively short event (8 days), and it is not expected that teams would be able to do all this, but it is important for teams to identify the good parts of their models as well as the parts that can be improved. Many teams shared their insights and provided potential things they could do in the future. In doing so they demonstrated one of the most important parts of the broader process of creating and analyzing mathematical models.

2.2 Modeling Issues

For this particular question, the primary population of interest has a fixed size. People move back and forth between the different compartments of the population and they do so on a time scale much faster than the broader changes in the population. The general modeling approach is based on the idea of conservation of mass, or in this case conserving the number of people in the population.

The models that were presented relied on this principle. Notably, some teams explicitly identified this important part of the modeling approach. The teams that noted this and demonstrated how their models conserved mass showed tremendous insight into an important and fundamental principle of mathematical modeling.

Another important issue associated with modeling is the role of choosing and exploring the parameters associated with the models. A number of teams examined their results and compared them over a range of values of some of the parameters they felt were important. This is an effective way to explore and examine a model. It allows us to determine the range and types of results that a model can provide.

One aspect that is related to the exploration of the parameters is to examine the sensitivity of a model to small changes in the parameters. Several teams explicitly examined and compared the results of their model after making small changes in one or more of the parameters. In doing so, these teams were asking important questions about how the broader behaviour of their models.

Finally, in addition to examining the different kinds of predictions a model can provide, some teams provided valuable insights into their modeling and the process of modeling. For example, a small number of teams noted the importance of the "anti-group." That is, they realized that in order to explore the dynamics of people who are more comfortable moving into a different social group based on an affinity for their mannerisms, there are also people who wish to move away from that group. Identifying the parts of a population that are not explicitly discussed and bringing them into the discussion of the model represents a significant level of insight into the modeling process.

2.3 Presenting Results

The insights and modeling efforts that a team brings into the construction of mathematical models is the first step. Another important part of the process is presenting and sharing results. The teams did a tremendous job in sharing complicated, technical information.

One of the difficulties in presenting information is the notation we use to condense a great deal of nuance into a compact form. Some of the documents included some nice models, but could be difficult to follow. Simply defining notation and making sure that the reader knows how to interpret individual terms can go a long way to making a document easier to follow.

Another aspect is how to convey a sense of how populations interact. For problem A, a number of teams were quite adept and providing graphical representations of how the different groups in their models interacted with one another. The graphs that were provided seemed much better than in previous years, and for the most part were a great aid in trying to understand the teams' work. Do not underestimate the impact of a simple diagram!

3 Problem B: Movement in Microgravity

Problem B was the least popular question in this year's event. The focus of this question is to predict the dynamics of the interactions between a small satellite and a small asteroid. The primary objective is to determine how to land and move a satellite on the surface of the asteroid, and the motivation comes from the landing of a Japanese probe on the asteroid Ryugu.

The outline of this section is the same as the previous section. First a description of some of the models is given. Next an overview of the issues that the teams examined is given. Finally, some general comments about modeling that arose for this question are given.

3.1 Models Used

The primary modeling approach for this question revolved around Newton's Second Law. Most of the teams made use of the conservation of momentum, and they developed a model based on the differential form, $\vec{F} = m\vec{a}$, of Newton's Second Law. A small number of teams considered the conservation of energy. A very small number of teams also made use of the conservation of angular momentum.

One of the difficulties for this problem was to define the full state of the system. This primarily includes the linear movement of the satellite including position as well as velocity. Those teams that also considered a broader system including the kinetic energy and the angular momentum were able to incorporate a more sophisticated sense of the dynamics of the system. Being able to integrate the larger system and implement the conservation of linear momentum, angular momentum, and kinetic energy is a non-trivial undertaking and is a substantial challenge for this question.

3.2 Modeling Issues

With respect to the development of the models for this question, the teams had to define a way to describe the state of the satellite. As mentioned above this depends on the extent of the dynamics incorporated in the model. To be able to construct a model, the teams had to clearly offer their assumptions as well as define the necessary variables.

In this situation defining and describing the variables is a challenge. For example, it was necessary to keep track of the size and orientation of the asteroid, the center of the asteroid, the height that the satellite is above the surface of the asteroid, and the momentum and position of the satellite. It is a difficult task to keep track of so many variables, and the notation could be difficult to follow.

Another difficult task is dealing with how the satellite might bounce as when it struck the surface of the asteroid. Almost every team struggled trying to determine how to model this behavior. The teams were also challenged with describing how an object bounced, and many teams were not able to address this difficult behavior.

3.3 Presenting Results

Presenting the model, its analysis, and the results was a significant challenge for this question. Surprisingly, the citations and references for the entries for this question were much shorter than for the other questions. The models relied on important and non-trivial notions of conservation, and the role of citations as well as references are a vital way to share the team's activities.

Another surprising part of the entries is that only a couple of the entries included a free body diagram of the satellite and asteroid. In any physical process relying on Newton's Second Law a free body diagram is absolutely vital in motivating and describing the resulting equations of motion. The lack of a free body diagram can make it difficult to read a paper and figure out how to interpret the system of equations that are stated.

Finally, another issue that seemed to be more prevalent for this question was a lack of captions for the figures. Moreover, a number graphs included axes that were not labeled, nor did they include units. Including units, labels, legends, and captions is a huge aid in trying to read and interpret the complicated technical information that is being shared in any report.

4 Problem C: Chemical Espionage

The large majority of teams taking part in the event this year chose to address the questions in problem C. I was surprised and expected that the question about the interactions between probes and asteroids would be more popular, because the potential ways to approach the problem would be clearer compared to the other questions. To my surprise more teams decided to examine this question, and to my delight the teams did a wonderful job in exploring these questions and made use of a wide variety of different approaches.

The outline of this section is the same as the previous sections. I will first provide a broad overview of the different models that were used. Next, some of the issues that came about from the team's efforts are discussed. Finally, some of the issues associated with presenting results are discussed.

4.1 Models Used

A wide variety of models were used for this question. The two most popular approaches were models based on logistic growth and models based on Lotka-Voltera type interactions. Several teams started with one of these kinds of approaches and then adapted to make use of the other. It is encouraging to see this kind of creativity in bringing together multiple, appropriate approaches in creative ways.

A small number of teams made use of compartmental models. At least one of these teams constructed a model similar to a Leslie population model type approach. In that model the different populations progressed through different stages which allows for the interactions within the age structure (egg, larvae, nymph, and adult) to be easily described.

With respect to the individual terms in the models, a few teams made use of a Type II (Holling's type) response function to approximate the rate of predation. In this particular case, the wasps require a substantial amount of time when engaged with a moth in order to parasitize the eggs. Given the way the two species interact this may be a more appropriate way to estimate the interactions between the two species.

Finally, at least one team made use of an interaction rate that introduced an Allee effect. This is an interesting approach, and it is not one that I expected to see. The motivation for the choice was not fully explained, though, and I am personally unaware of the appropriateness of this decision. Given the relatively large spatial distances required by the butterflies it may well be a good assumption.

In all these models one of the key aspects to consider was how to describe the state of the system. In the original paper, the authors state that the wasps are applying pressure for the butterflies to change their behavior. Most teams focused their efforts on modeling the dynamics between the two species without regard to sub-populations. Some teams, though, split the butterfly populations into two groups, and the two groups made use of different rates of use for the anti-pheromone. The models that these teams developed were more closely aligned with the final observations and questions raised by the authors of the original paper.

Surprisingly I did not see any team make use of disease type models such as an SIR model. When dealing with parasitic relationships it is not uncommon to treat the interactions like a disease. In this particular case, though, the biology is more reminiscent of more traditional predator-prey dynamics, and the models employed may be more appropriate.

4.2 Modeling Issues

One of the primary challenges faced by the teams for this question was to describe the state of the system. Most teams made use of a straight-forward predator-prey relationship. Given the basic idea of the question it was generally clear how to interpret the resulting model. However, the broader context involved the interactions of male and female butterflies with varying usage rates of an anti-pheromone.

Given that context, it was not always clear how the relationships between the males and females was being modeled. In many population modeling exercises the females in the population are modeled while ignoring the males. This is a rare case, though, in which the actions of the males have a direct impact on the whole population. A relatively small number of teams addressed this issue, and in doing so provided more insight into understanding some of the basic questions.

With respect to the analysis of the models, a large number of teams provided more analyses of their models. This is a wonderful development and represents a significant advancement that speaks well of the students' commitment as well as the efforts of their advisors. Many student teams were also able to present approximations of their models. The most common form was to see the approximations represented as a time series. Another common form was to be given graphs of the solutions in the phase plane. Best of all, a small number of teams provided both kinds of plots which allows for an ideal context to discuss the general behaviour of the systems they developed.

Another interesting development with respect to the analysis of the models this year is that a number of teams examined the fixed points of their models. Furthermore, several of those teams examined the Jacobian of their systems at those fixed points. Their examination of the linear stability at the resulting fixed points offered important insights into the general behavior of the systems they developed. This is something that was quite rare to see in previous years, and it is an exciting development to see student teams engaging in this vital aspect of the analysis of their mathematical models.

4.3 Presenting Results

From the context of the original question it was usually clear that a basic predator-prey system was being used by a team. Many teams also made assumptions that could be figured out, but a reader not familiar with the original question would likely have a difficult time following. When writing the reports it is important to identify the reading audience, and it should not be assumed that the person reading the report is familiar with the original question.

A related issue is that the underlying assumptions made by the team are not always clear. An explicit and clear discussion of the underlying system, motivations for decisions about a system, and the basic assumptions is a great aid to the reader. Making the background and assumptions clear can make the context clear to the reader and allows the reader to more easily understand the considerable efforts made by the team.

In terms of laying out the general assumptions there were two other positive developments this year. First, as mentioned in the description for problem A, a number of teams provided diagrams that demonstrated the assumed interactions between the different parts of the states being tracked in a given model. A simple diagram that shows the relationships between the different states is a huge help in understanding a team's model, and a number of teams made use of this simple but effective tool.

Another important advancement is that a number of teams provided a model with some analysis, but they then proposed either a new model based on their critical analysis or they offered suggestions on how to improve their model. The modeling cycle is one of the most basic and important parts of the modeling process. Each step of the cycle requires an honest critique to probe for the strengths and the weaknesses. This part of the process makes it possible to implement incremental improvements that will advance the goals of the team.

5 General Observations

Finally, some general observations are provided here. These are general comments that apply to all of the questions. The first is a note about copyrights. The second is about how to share code. The last is a remark about a difficult but important topic, scaling.

The first comment is about copyright. A large number of presentations included images without attributions. This is problematic from the point of view of copyright as well as ethical concerns. Deciding on whether or no the use of an image falls into the category of fair use can be a difficult decision, and I am not qualified to make a determination. It is important to be aware of the issue. It is vital, though, that whenever something is shared from another source that it be cited and a reference provided.

The second issue is about how to share code. There were several presentations that included snippets of computer code. It is good to include code in a separate appendix of a report. The inclusion of code in a presentation, though, can be distracting. It is good to discuss the techniques and approached employed in the code, but in a presentation the actual code is not necessary.

The final issue is about scaling. This can be a very difficult topic when first seen. It is not clear why it is done or what it even means. The techniques used require experience and a broader understanding of the purpose and goals, so it can be a deeply disconcerting exercise when first performed. At the same time, it is a central part of modeling, and creating a non-dimensionalized system can reduce a complicated model into a system with fewer parameters whose analysis is more straight forward.

While it is a difficult topic, scaling is one that advisors may want to consider sharing. Teams are creating more complicated models, and a number of teams are delving into deeper explorations of how models depend on various parameters. Being able to explore with respect to the innate scales associated with a model can be a huge step forward in how to analyze a given model.