www.simiode.org

SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION

Atmospheric CO_2

Jakob Kotas
Department of Mathematics
University of Portland
Portland OR USA

STATEMENT

Suppose that the average earth temperature as a function of the level of atmospheric CO_2 is modeled using the following first-order ordinary differential equation (ODE):

$$\frac{dT}{dt} = -0.005T^3 + 0.12T^2 - 0.675T - 0.8 + 0.004c$$

where T is temperature in degrees Celsius above a constant reference temperature, which we refer to as the baseline. T is taken to be a function of time t in years, and c is the global average atmospheric CO_2 level in parts per million (ppm).

During the last 800,000 years, and before the industrial revolution, global average atmospheric CO_2 fluctuated between about 180 and 280 ppm.[1] Let us look first at the case where $c = 200$ ppm, which was a typical level during the ice ages.

1. Let us say that during a time when $c = 200$ ppm, there is a major global event that heats the earth by 0.25°C in a very short period of time (months to years), such as a strong El Niño, or a peak of the solar irradiance cycle. Use a numerical solver, such as Matlab's `ode45`, to solve for $T(t)$ after this event. (In other words, take $T(0) = 0.25$.) What temperature does the earth approach after some time after the event? How long does it take for the earth to cool down to within 0.1°C of its baseline?

2. Draw a phase plane with $\frac{dT}{dt}$ on the vertical axis and T on the horizontal axis. Identify the equilibrium points: the places where $\frac{dT}{dt} = 0$. Find the stability of the equilibrium points by considering values of T slightly above and below the equilibrium point. Argue why your answer to Question **1**

makes sense in light of this.

3. Now consider another time when $c = 275$ ppm, a value seen historically during interglacial warm periods. Again draw the phase plane and identify the equilibrium points, along with their stability. Use a calculator to find numerical values for the equilibrium points. What is the meaning of these numbers?

4. Again, use a numerical solver to solve for $T(t)$ with $T(0) = 0.25$, but now use $c = 275$ ppm. How does this result differ from your answer to Question **1**? Does this make sense? Would a major global event that heats the earth by 0.25°C still mean that $T(0) = 0.25$? Find the answer again with a different initial condition that reflects such a situation, and confirm that the result matches your intuition.

5. In 2018, the global average atmospheric CO_2 was about $c = 407$ ppm. Draw the phase plane again, find all equilibrium points using a calculator, and determine the stability of each. How is this different from your answers to Questions **2** and **3**?

6. Imagine that a rare and catastrophic short-term warming event happened today and that it heated the earth by much more than 0.25°C; let's say it heated the earth by an amount b°C (above the lowest stable equilibrium point found in Question **5**). Is there a value of b beyond which the earth would not be able to recover back to temperatures within a few degrees of baseline? Find the value of b using a calculator. If such an event happened, what would happen to the earth's temperature? Where would it settle? Let us call this point "cataclysmic heat."

7. You should see a pattern in your phase plane graphs from Questions **2**, **3**, and **5**. If c were to keep increasing, is there a point beyond which the only stable equilibrium point is cataclysmic heat? Use a calculator to find this value of c. What would happen if c were to increase above this level? Some argue that this is what would happen if the ice at earth's poles melted completely.

8. Let's say that CO_2 levels rose briefly above c and the temperature reached cataclysmic heat. If the governments of the world, at this point, realized what was happening and developed a way to make CO_2 levels drop back below c again, would the earth be able to recover from the cataclysmic heat situation?

REFERENCES

[1] Earth System Research Laboratory-Global Monitoring Division. 2013. *CO_2 at NOAA's Mauna Loa observatory reaches new milestone: tops 400 ppm*. Available online: `https://www.esrl.noaa.gov/gmd/news/7074.html`. Retrieved Dec 29, 2019.