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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
FALLING IN WATER

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STATEMENT

In the iconic differential equation text [1, pp. 43-48], there is a section on modeling a falling canister of nuclear waste in the ocean to determine the velocity that the canister might have upon striking the bottom of the ocean. This is due to concern that the canister might break open and the lead containment surrounding the nuclear material might fail.

Figure 1. Canister used in experiment.

Activities

1. Model the fall of our own canister in a cylinder of water as shown in Figure 1 in our own small little "ocean" as we place it in the water. Figure 2 shows you our setup for making the video posted on YouTube [4] from which we took data in Table 1.

   You will need to capture your own data using the set up depicted in (see Figure 2) from the YouTube video [4]. This video was captured by an iPad 5 and then the app SlowMo was used

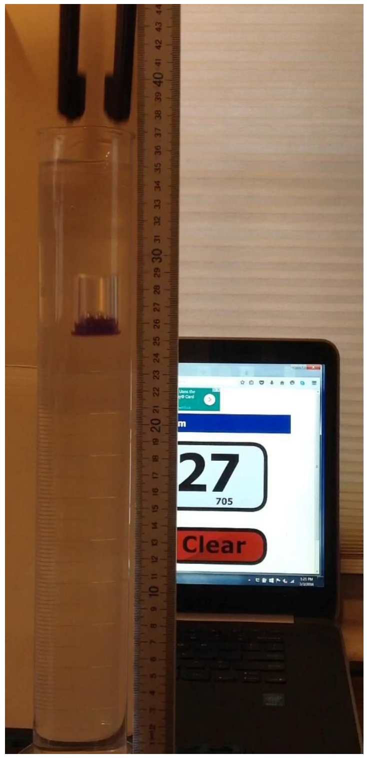

Figure 2. Experiment set up with cylinder of water and meter stick with canister released and falling in the foreground and laptop computer with digital clock for timing in the background. This image was taken with an iPad as was the YouTube video [4].

to slow the video down to 25% of actual speed for ease in collection of data by stopping and starting the video.

You should produce two models, one with resistance proportional to velocity and one in which resistance is proportional to velocity squared. Be sure, in both cases to consider all forces acting on the canister as it falls.

2. You might consider a more general situation in which resistance is proportional to velocity to some unknown power?
3. Compare your models in some manner. We offer one method called the Akaike Information Criterion (AIC) [2, p. 138].

Akaike Information Criterion

The Akaike Information Criterion (AIC) offers a way to compare different models of the same phenomenon. Variables used for the number or score which is the AIC are: p, number of parameters to fit; n, the number of observations; and RSS, the minimum residual sum of square errors (or just sum of square errors, SSE). The actual formula for AIC is given in (1):

$$AIC = 2(1+p) + n\log\left[\frac{\text{RSS}}{n}\right]. \qquad (1)$$

NB: when RSS or SSE is small then the log function is negative.

Smaller (even negative) values of AIC represent a higher level of statistical support for the corresponding model. A difference of 2 in the AIC's is needed to suggest a significant difference between models, with the smallest AIC signifying the best model - in terms of balancing ACCURACY (RSS) and COMPLEXITY (p). However, if all AIC's are close (within 2) to each other then one cannot distinguish which model is better.

APPENDIX

Table 1 shows the formatted data taken from an earlier video [3]. For this data set the mass of the canister with the steel balls was 15.0 g as it had two more steel balls than the canister used in [4], but the dimensions of the canister were the same, height of 3.6 cm and radius of 1.1 cm.

t	0.00	0.26	0.4,	0.56	0.70	0.90	1.00	1.19	1.26	1.40	1.52	1.66	1.80	2.06
$d(t)$	0.0	1.1	2.3	4.1	6.2	8.5	10.4	13.	14.1	16.2	18.7	20.4	22.5	26.8

Table 1. Data on falling canister from video. t is time in seconds while $d(t)$ is depth of fall in cm at time t.

REFERENCES

[1] Braun, M. 1978. *Differential Equations and Their Applications, Second Edition*. New York: Springer Verlag.

[2] Ledder, G. 2013. *Mathematics for the Life Sciences*. New York: Springer.

[3] SIMIODE. 2016. Canister Falling in Water, Early Version. `https://www.youtube.com/watch?v=40MCsVJD1FI`. Accessed on 17 April 2016.

[4] SIMIODE. 2016. Canister Falling in Water. `https://www.youtube.com/watch?v=oNoPw9GFf2g` or at SIMIODE Website `https://www.simiode.org/resources/1703/download/CanisterFallInWater.mov`. Accessed on 17 April 2016.